Endeavouring to teach mathematical problem solving from a constructivist perspective: The experiences of primary teachers

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Abstract

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The Irish primary mathematics curriculum is based upon a constructivist philosophy of learning. As constructivism is a theory of learning and not teaching, it requires teachers to identify the implications for teaching. This study describes the experiences of five primary teachers as they attempt to explore mathematical problem-solving from a constructivist perspective with primary school children in Ireland. The key question upon which the research is based is: to what extent will an understanding of constructivism and its implications for the classroom impact on teaching practices within the senior primary mathematical problem-solving classroom? Constructivist theory has evolved from early learner centred education initiatives but the impetus for the constructivist movement of the twentieth century can be attributed to Jean Piaget and Lev Vygotsky. Several perspectives on constructivism have evolved with the emergent perspective on constructivism being central to the Irish primary mathematics curriculum.

Following the involvement of five primary teachers in a professional development initiative involving constructivism in the context of mathematical problem-solving, case study was employed to record the teachers' experiences and the experiences of their students as they engaged in a constructivist approach to problem-solving in the classroom. These case studies reveal primary teachers' interpretations of constructivist philosophy and the implications for teaching in a primary mathematics classroom. The study identifies effective strategies for exploring mathematical problems from a constructivist perspective. The study also illuminates the difficulties in making the transition from utilising traditional methods of teaching mathematics to employing those teaching strategies that reflect constructivist philosophy.

DECLARATION

I hereby declare that this thesis represents my own work and has not been submitted, in whole or in part, by me or another person for the purpose of obtaining any other qualification.

Signed: _____

Date: _____

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DES	Department of Education and Science
ERC	Educational Research Centre
NCCA	National Council for Curriculum and Assessment
NCTM	National Council of Teachers of Mathematics
OECD	Organisation for economic Co-Operation and Development
PCSP	Primary Curriculum Support Programme
PISA	Programme for International Student Assessment
PPDS	Primary Professional Development Service
RME	Realistic Mathematics Education
TIMSS	*Trends in International Mathematics and Science Study*
USSR	*Union of Soviet Socialist Republics*

Chapter 1

Introduction

1.1 Introduction

In this millennium there is an increasing demand for highly skilled trained professionals in societies that have experienced a seismic shift towards a knowledge focus. Hence, mathematical literacy is regarded as valuable; it is essential in fields of science, economics, engineering, and psychology. However, what is mathematical literacy and, more importantly, how can it be achieved? The Organisation for Economic Co-Operation and Development (OECD) have defined mathematical literacy has as 'an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen' (OECD, 2003:24). The fundamental implications for mathematics educators are clear. Mathematics must make sense to the individual, it must be grounded in his/her experiences and any mathematics education must build upon these prior experiences.

Achieving mathematical literacy has long been a struggle for many students, perhaps because mathematics may be perceived as abstract and difficult, especially in our schools. This is evidenced by the number of students that study mathematics to higher level and beyond. According to the State Examinations Commission, only 17 per cent of students who sat the Leaving Certificate in 2008 undertook higher level mathematics (Government of Ireland, 2009a). When mathematics is discussed by adults, inevitably experiences at school are recalled and often conversations revolve around the rules and theorems memorised for the purpose of state examinations. Mathematics remains a subject that many fear, perhaps because of teaching practices that render the subject

mysterious. Direct, rule-oriented mathematics teaching practices often fail to place mathematics in context for the student. These practices, termed 'parrot math' (Van de Walle, 1999) persist today but attempts at reculturing mathematical classrooms have resulted in a 'mélange' (Cohen, 1990:311) of teaching practices encompassing both traditional and reform-oriented instructional methods. Darling Hammond (1996) warns the major challenge of this century will be moving on from traditional notions of what qualify as teaching.

> If we want all students to actually learn in the way that new standards suggest and today's complex society demands, we will need to develop teaching that goes far beyond dispensing information, giving a test and giving a grade. We need to understand how to teach in ways that respond to students' diverse approaches to learning, that are structured to take advantage of students' unique starting points, and that carefully scaffold work aimed at more proficient performances. We will also need to understand what schools must do to organise themselves to support such teaching and learning. (Darling-Hammond, 1996:7).

However, what might alternative forms of teaching resemble?

Any attempt to teach mathematical content to students without seeking to make it relevant to them has little hope of succeeding in the long term. Constructivist theory offers an alternative to traditional methods of teaching. Constructivism shares the metaphor of carpentry, architecture or construction work. Von Glasersfeld (1989:182) explains that 'knowledge is not passively received but actively built up by the cognising subject'. Constructivist theory is central to our current curriculum and was also central to the curriculum of 1971. A constructivist approach to the teaching of mathematics involves teaching for understanding, holding that the current knowledge and experiences of pupils are the foundation blocks for future constructions that constructivists try to enable children to build. An individual with such a mathematical background should be well placed to becoming mathematically literate.

A constructivist approach to teaching can be effective, but are we sure of its implications for the classroom? We as teachers need to have a good understanding of it. The basic principle of constructivism is that children construct their own knowledge. This is a broad sweeping principle and the implications for the traditional classroom are

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far reaching. Do we have enough evidence upon which to build solid curricula, to prepare primary teachers for employing constructivist principles in the classroom, and to move away from what has characterised the very essence of teaching for decades? Research would suggest we have not moved far in a constructivist direction to date even though it has been at the heart of the primary mathematics curriculum since 1971 (Surgenor, Shiel, Close and Millar, 2006; Windschitl, 2002). This is because as Airsian and Walsh (1997) reveal, constructivism is a theoretical framework that broadly explains the human activity of knowing, but, unfortunately, offers teachers very little detail in the art of teaching. Therefore, it is a theory of learning that must be somehow be translated into a theory of teaching.

Proponents of educational reform view the process of getting to know mathematics as a social endeavour that happens during the interactions within the classroom (Ball, 1993; Bauersfeld, 1995; Cobb, Yackel, and Wood 1993; Lampert, 1990). This is an emerging perspective on constructivism reflected in the current mathematics curriculum. Such interactions are characterised by having students think, talk, agree, and disagree about mathematics that is relevant to them. Both the Principles and Standards for School Mathematics (NCTM, 2000) and the Primary Mathematics Curriculum (Government of Ireland, 1999a; 1999b) recommend that, rather than explaining and demonstrating, teachers should move towards a non-traditional way of teaching in the classroom by allowing students to explore meaningfully their own mathematical ideas, to express them, and to take account of the thinking offered by others. Fullan (1993) and Joseph, Bravmann, Windschitl, Mikel and Green (2000) explain that effective forms of constructivist teaching depend on nothing less than the reculturing of the classroom, but that the features that make constructivist classrooms effective complicate the lives of teachers, students, administrators, and parents. Teaching contexts, teacher characteristics, teacher thinking, and their interactions are influential factors in attempts to implement classroom reform (Gess-Newsome, Southerland, Johnston and Woodbury, 2003). Many reform initiatives have arrived at the classroom doors of teachers but Cuban (1988) noted that reforms that seek to change fundamental structures, cultures and pedagogies are difficult to sustain and progress. Enacting classroom practices that support discourse in the mathematics classroom poses challenges for teachers since they bear little resemblance to their current practices (Nathan and Knuth, 2003). Any reform initiative needs to be well supported in an

effort to sustain it in the long run. Reform in mathematics education was the subject of high profile debate only recently in the United States, and became known as the 'math wars' (Schoenfeld, 2004).

The 'math wars' (Schoenfeld, 2004) raged in the United States after the publication in 1989 of the Curriculum and Evaluation Standards for School Mathematics, a predecessor of Principles and Standards for School Mathematics by the National Council of Teachers of Mathematics. Principles and Standards for School Mathematics (NCTM, 2000) explains that students must experience mathematics for understanding and learn by building new knowledge actively from experience and prior knowledge. Effective mathematics teaching requires teachers to understand what students know and need to learn, and that students need to be challenged to support them in their learning (NCTM, 2000).   This is what has been termed, reform mathematics.   Reform mathematics concerns how children learn and how they achieve the goals of the curriculum. On one side of 'the war' are those who strongly believe that children need to learn the basics, and on the other are those who believe in the message of the standards or reform mathematics. What are the basics? Simply, they are considered to be an understanding of simple computation, knowledge of formulae and their application, basic fact mastery, and mastery of measurement conversions (Van de Walle, 1999), in effect, content. Both sides of the arguments have taken some extreme positions. For example, those vehemently concerned with the mathematical basics have highlighted skills not reflective of the needs of today's society while some reformers have failed to emphasise many valid content objectives. In this exchange people have forgotten both about the appropriateness of reform initiatives and the importance and relevance of some of the content approach to mathematics teaching.

Irish primary teachers have been engaged in reform in Irish primary classrooms for the past ten years. We know a lot about teaching and learning in the primary school and our curriculum has undergone reform and evaluation (NCCA, 2008), but to what extent is this knowledge and reform impacting upon teaching practices within the primary mathematics classroom? Worryingly, particularly in relation to mathematics education, the National Council of Curriculum and Assessment's Review of the Primary School Curriculum (NCCA, 2008a) has revealed that teachers still feel challenged by methods of teaching, particularly group teaching, espoused by the Primary Curriculum. Various

reports and research conducted since 1999 have revealed that Irish primary students can perform basic mathematical skills quite well and that they know their mathematical facts, but, at second level, 15 year old students compare poorly with other countries in relation to higher level mathematical processes such as reasoning, analysing, solving problems, and analysing solutions (Eivers, Shiel and Cunningham, 2007; Mullis, Martin, Beaton, Gonzalez, Kelly and Smith, 1997; Surgenor, Shiel, Close and Millar, 2006). This is typical of students who have come through an education system that places significant emphasis on direct instruction and less emphasis on alternative forms of instruction. Gash (1993) reported that even though constructivist principles underpinned the curriculum introduced in primary schools in 1971, teachers continued to utilise didactic and teacher directed methods of teaching. We need an examination of teaching practices, a snapshot of a typical teacher's approach to mathematics education and in particular their teaching and exploration of higher level mathematical processes, to realise how we might successfully implement alternative approaches to mathematics education.

It is interesting to note that, at second level, Project Maths was launched in September 2008. This development places a greater emphasis on student understanding of mathematics concepts, with increased use of contexts and applications that will enable students to relate mathematics to everyday experience (NCCA, 2008b). This initiative is essentially constructivist in orientation. From a primary perspective, research into mathematics education teaching practices in Ireland is limited, even though significant changes have been made to the curriculum in the primary school in the last number of years. Students experience formal mathematics for the first time at primary level and teachers, therefore, will foster and shape any attitudes students will develop towards mathematics at primary level. Hence, research into the mathematics teaching practices of primary teachers is timely.

This research attempts to investigate typical teachers' exploration of constructivist practices in the fourth, fifth and sixth classes of the primary classroom following their engagement with constructivism and mathematical problem-solving from a constructivist perspective in a professional development initiative delivered during the autumn of 2007. The key question upon which the research is based is: to what extent will an understanding of constructivism and its implications for the classroom impact

on teaching practices within the senior primary mathematical problem-solving classroom? Implicit in this question is the professional development of teachers. Following their engagement with a course in constructivism and mathematical problem-solving from a constructivist perspective, six primary teachers consented to being the subjects of a case study, the primary research methodology utilised for the purposes of this research. Participating teachers allowed their teaching of mathematical problem-solving from a constructivist perspective to be observed, and both teachers and their students consented to engaging in semi-structured interviews and group interviews at stages throughout the research. We have been engaged in a process of reform in the Irish primary situation since the introduction of the Primary School Curriculum (1999). It is timely perhaps to examine how teachers who teach in ordinary schools, under ordinary conditions interpret that process.

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1.2 Outline of dissertation

Chapter Two presents a summary of the literature that informed this research. The literature examines and debates constructivism, mathematics education and mathematical problem solving. It provides a historical overview of educational developments and key stages in developments in education that have shaped and continue to shape the mathematics curriculum and the teaching of mathematical problem solving. The primary focus of this chapter is to discuss the influence of constructivist principles on curriculum, teaching and learning, and their consequences for the mathematical problem solving classroom. The chapter discusses mathematics education in Ireland, and examines results and achievements of students in comparison with their European and international counterparts in an effort to describe why this study is particularly necessary in the field of mathematical problem-solving education.

Chapter Three justifies the research methodology employed to investigate the research question. This chapter presents a rationale for undertaking the research. Following an examination of the principles of successful professional development, it describes in detail the professional development initiative participating teachers engaged in. It provides relevant information on research sites, settings and participants. It presents a description of the research tools utilised to gather data for the purposes of the research

and outlines the philosophical underpinnings of case study and the key aspects of successful case study methodology.

Chapter Four presents the data gathered throughout the course of the research from each individual case in significant detail. This data was gathered in the classroom as the individual teacher engaged their pupils with mathematical problem-solving from a constructivist perspective, and in semi-structured interviews with the participating teachers throughout the period of the research. Case study data is presented in relation to each participant to give a complete depiction of the participant's experiences as they engaged in the research.

Chapter Five presents an analysis of the data. Following examination of all cases, common themes emerged; these themes are discussed and initial implications arising from them are considered. This chapter presents an analysis of the cases on an individual basis. From the series of semi-structured interviews with both students and participating teachers, resulting in audio evidence and documentary evidence, a number of themes emerged that revealed participants' approach to the employment of constructivist teaching practices in mathematical problem-solving lessons, and placed their approach to mathematical problem-solving in context.

Chapter Six reflects on the findings of this qualitative research in the light of research in the field of mathematics education and constructivism, and emphasises the contribution of this study to the promotion of constructivist teaching methods in the mathematical problem-solving classroom.

Chapter Seven presents the findings of this research, draws conclusions, and makes recommendations for theory, practice, future curriculum development, curricular support initiatives, and initial teacher education programs.

Chapter 2

Literature Review

2.1 Introduction

The central focus of this thesis is the investigation of critical teaching issues, involving the engagement of senior primary school pupils with mathematical problem solving from an emergent constructivist perspective. Therefore this chapter focuses on an analysis of constructivism from various perspectives which have influenced and continue to influence curricular developments. As this thesis specifically examines problem solving, throughout the review of literature, key issues related to the teaching of mathematical problem solving are presented. This chapter also provides a historical overview of constructivism which has significantly shaped and continues to influence curriculum. In particular, this chapter looks at the Piagetian basis of Curaclam Na Bunscoile (Government of Ireland, 1971) and examines how the current Mathematics Curriculum (Government of Ireland, 1999a; 1999b) is founded on constructivist principles specifically emphasising a social element.

The learner centred education movement has been a significant influence in the development of curricula, and on approaches to teaching, learning and assessment, and is integral to an examination of constructivist theory. The early Sumerians and some of the greatest educators of the nineteenth and twentieth centuries, including Jean Jacques Rousseau and John Dewey, have championed child centred education or learner centred education and continue to have a major influence on educational developments. The literature of Jean Piaget and Lev Vygotsky are central to examining constructivism from a historical point of view. Piaget has been credited with giving the impetus to the constructivist movement of the twentieth century, and Vygotsky further developed the constructivist theory by highlighting the intrinsic importance of the social experience. In the Irish context, both the work of Piaget and Vygotsky are very evident in the principles of the Mathematics Curriculum (Government of Ireland, 1999b).

Constructivism is particularly examined from the radical, social and emergent perspectives. Radical constructivism is centred on the experience of the individual alone. Social constructivism is distinct from radical constructivism in that it emphasises the impact of the social environment on the learner and on his/her learning. The latest development involving constructivist theory has been the emergent perspective. It acknowledges the significance of both the social context of the learner and his/her own personal experiences. This particular perspective is what is key to this research and it has it's foundations in the work of American researchers of the early 1990s (Cobb and Yackel, 1996). It evolved from both the psychological and social perspectives and it will be argued is the most practical version of constructivism to be employed in the primary classroom.

This chapter also discusses mathematics education in Ireland, and compares the performance of Irish students in relation to mathematical problem-solving with their European and international counterparts. This section reveals the necessity of researching the teaching of problem-solving using a constructivist framework from an Irish perspective. Finally, this chapter examines educational developments in Ireland, and describes specific current research in the area of mathematics education.

2.2 Constructivism: An introduction

Constructivism will be examined from three distinct perspectives namely the radical, social and emergent perspectives but it is first of all necessary to examine the broad principles of constructivism as they relate to the classroom. From the outset it is necessary to state that, although it may influence teaching, constructivism is a theory of learning and not a theory of teaching (Wolffe and McMullen, 1996). It is an epistemology, a learning theory that offers an explanation of how we learn. The challenge therefore is translating this theory of learning into one that is usable for teaching. Emergent constructivist theory suggests that humans generate knowledge from their experiences and interactions with one another (Matthews, 2000). In a constructivist classroom, the learner should play an active role in the learning process in an environment designed to support and challenge him/her. Learning activities in constructivist settings are characterised by active engagement, inquiry, problem solving and collaboration with others. Constructivist principles are woven into many curricula worldwide, yet the implications for classroom teaching are still evolving. The teaching

of mathematics has long been subjected to debate arising primarily from different opinions as to what constitutes effective mathematics teaching. Constructivism plays a central role in this debate. Much literature describes how teachers could approach mathematics from a constructivist perspective, and this will be presented in the literature review. In adopting a constructivist approach to mathematics education teachers must focus on meaning and understanding in mathematics and encourage autonomy, independence, and self direction (Petersen, 1988). Adopting such an approach to mathematics education is difficult because it differs significantly from long held beliefs about what constitutes sound mathematical explorations and quality mathematics teaching. Desforges and Cockburn (1987) and Nathan and Knuth (2007) explain that a classroom based upon active learning, problem solving, and small group work can be difficult to establish,

> Tasks with higher level cognitive demands increase the pupils' risk and the ambiguity involved in engagement and thus alter the commonly established exchange rate in classrooms – that of an exchange of tangible rewards for tangible products. Pupils like to know where they stand. For this reason, tasks demanding higher order thought processes are resisted or subverted by pupils. Resistance puts co-operation at risk. Teachers are lured into or connive at subversion and higher-level task demands are frequently re-negotiated in the direction of routine procedures (Desforges and Cockburn, 1987).

The path to universally accepted approaches to mathematics teaching and education has not been easily identified. For example, prior to the publication of the Cockroft Report in the United Kingdom in 1982, a back to basics approach to mathematics education was recommended to teachers of low achieving students (Cockroft, 1982). The Cockroft Report (Cockroft, 1982) strongly rejected this approach and claimed 'an excessive concentration on the purely mechanical skills of arithmetic for their own sake will not assist the development of understanding in these other areas' (Cockroft, 1982:278). A similar argument took place in the US during the 1990s (Schoenfeld, 2004). Therefore, perspectives on approaches to teaching mathematics differ but what remains constant in current curricula (NCTM, 2000; Government of Ireland, 1999a; 1999b) is that teachers must ensure children become mathematically empowered.

2.3 Mathematical problem solving

What is mathematical problem solving? Mathematicians are not unanimous in their conception of what mathematical problem solving is. Stanic and Kilpatrick (1988) elaborate on three different themes associated with mathematical problem solving ranging from mathematical problem solving as context to mathematical problem solving as art. In these themes, the conception of mathematical problem solving varies from mathematical problem solving being utilised in the service of other curricular goals to mathematical problem solving being at the very heart of mathematics. According to Schoenfeld (1994), there is one particular mathematical point of view regarding the role that problems have in the lives of those who do mathematics. This unifying theme is that the work of mathematicians is solving problems. Halmos (1980: 519) explains that a mathematician's main reason for existence is to solve problems, and that, therefore, what mathematics really consists of is problems and solutions (Halmos, 1980: 519). Therefore, mathematical problem solving is at the heart of any mathematicians work and to become a mathematician, one who discovers, conjectures, tests and proves one must become a problem solver. For this to happen, students must therefore engage in solving real problems in classrooms to become real problem solvers Halmos (1980).

Before engaging in an examination of the varying perspectives on constructivism, it is necessary to have an initial understanding of how mathematical problem solving is linked to the constructivist theory of learning and indeed the importance of mathematical problem solving. It is quite difficult to draw a distinction between the teaching of mathematical problem solving from a constructivist perspective and engaging children in mathematical problem solving as espoused by the Irish Mathematics Curriculum (Government of Ireland, 1999). Literature suggests that doing mathematics or problem solving is 'reaching the stage at which one is producing more of that stuff by oneself or in collaboration with others' (Scheonfeld, 1994) which, in itself, is innately constructivist. To elaborate, Schoenfeld (1994:58) suggests that although the result of doing mathematics may be 'a pristine gem presented in elegant clarity', the 'path that leads to that polished product is most often anything but pristine, anything but a straightforward chain of logic from premises to conclusions' (Schoenfeld, 1994: 58). This path is problem solving; in doing mathematics, children

engage in processes, such as reasoning, justifying and explaining, before reaching mathematical conclusions. This path is inherently constructivist because in problem solving by testing ideas, examining hypotheses and formulating solutions students are truly engaged in learning from a constructivist perspective. They are constructing understandings, generating knowledge often in the company of others. Many mathematical problems are considered too big for individuals to solve in isolation and this necessitates collaborative work which is an important aspect of learning from an emergent constructivist perspective. Mathematical problem solving cannot be construed merely as knowledge to be received and learned. The very essence of problem solving is the process of making sense of particular phenomena.

Francisco and Maher (2005), in a longitudinal study investigating the conditions for promoting reasoning in problem solving, state that 'providing students with the opportunity to work on complex tasks as opposed to simple tasks is crucial for stimulating their mathematical reasoning (Francisco and Maher, 2005:731). This is why problem solving plays a central role in current curricula including the Primary Mathematics Curriculum (Government of Ireland 1999a; 1999b) and the Principles and Standards of School Mathematics (NCTM, 2000). Curricular perspectives on mathematical problem solving reflect, very much, the notion that mathematics is about sense making (Schoenfeld, 1994). Current curricula suggest that students should be engaged in solving mathematical problems. For example, Principles and Standards for School Mathematics explains that 'students should have frequent opportunities to formulate, grapple with and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking' (NCTM, 2000: 52). The Primary Curriculum (Government of Ireland , 1999a: 35) states that problem solving experiences should develop in children 'the ability to plan, take risks, learn from trial and error, check and evaluate solutions and think logically. According to the mathematics curriculum, Government of Ireland, 1999a; 1999b), discussion and acceptance of the points of view of others are central to the development of problem solving strategies. Both curricula are constructivist, and as constructivism is de-constructed and debated in this chapter, it will become clear that exploring mathematical problem solving by engaging students in constructivist practices clearly makes sense. The Irish Primary Curriculum of 1971 (Government of Ireland, 1971) was built upon Piagetian principles and was followed by the Primary Mathematics

Curriculum of 1999 both incorporating Piagetian and Vygotskian principles. Both Piaget and Vygotsky are central to the birth of the modern constructivist movement and therefore both current and past curricula are closely linked with constructivist philosophy.

2.4 Constructivist theory

Before examining constructivism from various perspectives, it is prudent to look at key historical movements in which the seeds of modern constructivist philosophy were sown. In this section, particular emphasis is placed on movements in education that identify shifts away from rote learning and traditional methods of instruction, towards more significant student engagement with instructional material and learning. Although the timeframe is significant, it provides an appropriate introduction to the discussion of various constructivist perspectives.

2.4.1 Learner centred education

Early progressive movements championed child centred and learner centred approaches to education and advocated much the same instructional philosophy as constructivism does today. Since the dawn of time, education that focuses on the learner has been in evidence. It can be traced back to the time of the Sumerians and the development of written language some 5,000 years ago. Early teachers such as Confucius and Socrates emphasised the development of character, citizenship and the individual. Confucius believed that every person should strive for the continual development of the self until such time as excellence is achieved (Ozmon and Cramer, 2002). The following is a brief history of child centred and learner centred approaches to education.

The Advancement of Learning (1605) by Francis Bacon emphasised a shift away from rote learning and deductive reasoning toward a more comprehensive engagement with the world (Gould, 2005). During the sixteenth century, Francis Bacon (1561-1626) emphasised that we should begin our thinking with questions and end with certainties, rather than begin with certainties and end with questions. Bacon explained that our thinking is limited by what others believe, lack of experience, unclear language, and the influence of religion and philosophies (Henson, 2003). Bacon insisted that learning

should focus on problems and then make assumptions, therefore considering, all possibilities. Also, during the seventeenth and eighteenth centuries, John Locke (1632-1704) and Jean Jacques Rousseau (1712-1778) wrote extensively on education. Locke compared the mind of a child to a blank slate, 'Tabula Rasa'. According to Locke, the only way to fill the mind of the individual was for the individual to have many experiences upon which he must reflect. He asserted that at birth the human mind is a blank slate, empty of ideas. We acquire knowledge, he argued, from the information about the objects in the world that our senses bring to us (Aaron, 1971). We begin with simple ideas and then combine them into more complex ones. Locke believed that individuals acquire knowledge most easily when they first consider simple ideas and then gradually combine them into more complex ones. He believed that a sound education begins in early childhood, and insisted that the teaching of reading, writing and arithmetic be gradual and cumulative (Atherton, 1992).

Rousseau believed that education must be based on experience. He explained that children must be allowed to develop naturally and be free from outside influences. Rousseau believed in the educative power of nature in developing the child. This is explored in his treatise Emile, translated by Barbara Foxley in 2006. Here, Rousseau (2006) stresses the importance of developing our personal ideas in order to make sense of the world in our own way. He encourages Emile to reason his way through to his own conclusions, stressing that he should not rely on the authority of the teacher. Throughout his treatise, instead of being taught other people's ideas, Emile is encouraged to draw conclusions from his own experience (Rousseau, 2006).

Rousseau divides the development of the individual into five stages. Stage One concerns the period from infancy to the age of two and focuses on the liberation of the child from others. During the 'Age of Nature', from ages two to twelve years, Rousseau emphasises the development of the physical qualities of the human being and the senses, and remains unconcerned with the development of the mind (Bloom, 1991). During 'Pre-Adolescence', from age twelve to fifteen years, Rousseau states that the child is developing at a pace far quicker than he/she is able to deal with. During this stage of development Rousseau suggests that the individual should be supplied with Daniel Defoe's Robinson Crusoe, so as to learn the need for self-sufficiency which he saw as a paradigm for the beginning of the development of the mind. During the stage

of 'Puberty', from ages fifteen to twenty-one, Rousseau states that the individual is ready to cope with moral issues, religion, and the troubles of adolescence. In the final stage of development, 'Adulthood', Rousseau focuses on the individual's relationships with others (Bloom, 1991).

Rousseau dramatised his perspective on education through Emile and brought it to a wider audience. He focused on power and control over the child's environment. He stressed that the more successful the educator was at controlling the environment, the more successful the education of the individual would be. His philosophy is centred on harmony and the development of the whole person. This would be achieved by taking control of the individual's education and environment, based on an analysis of the different physical and psychological stages through which he passed from birth to maturity (Bloom, 1991). In Emile, Rousseau argues that the momentum for education comes from the maturation of the learner and that it is the function of the educator to provide for this momentum (Rousseau, 2006). Rousseau explained that every mind has its own form and, therefore, that education should progress at the level required by the individual (Bloom, 1991). As outlined above, it was not until the age of twelve that the mind of the child would be ready for literature and development. His was a radical perspective that prefigured elements of constructivism. Inherent in constructivism is the interaction between the individual and his environment. Constructivists assume that an individual must reconcile problems in his/her understanding by attempting to associate them with experience, much like Rousseau's Emile.

One who began to see how community can impact on the development of the individual was John Dewey. John Dewey (1859-1952) spent most of his life philosophising about how the child must be educated at both psychological and social levels. He believed that education must begin with understanding how the child's capacities can be directed to help the child succeed in the community (Dewey, 1956). Unlike Rousseau, Dewey placed emphasis on the social setting and the child's involvement in social activity. He advocated that the school should be a microcosm of the community. In his laboratory schools, such a philosophy was born. Laboratory schools used problem-solving to a significant extent, and teachers ensured that each experience motivated the child toward further exploration. The interest of the pupil was of paramount importance; the experiences of pupils came from within themselves. John Dewey intended that

educative experiences would be social, be connected to previous experiences, be embedded in meaningful contexts, and be related to the students' developing understanding of content (McDermott, 1981). Pestalozzi and Froebel opened schools based on a learner centred curriculum, emphasizing the development of the whole child, physical, emotional and mental (Silber, 1965; Liebschner, 2002). These educational experiences were established to nurture positive self-development, free of risk and fear.

Many people established schools based on child centred theories. Colonel Parker during the 1800's endeavoured to implement learner centred education. He led reforms in Quincy, Massachusetts, and at Chicago's Cook County Normal School based, in part, on the child centred theories of Rousseau, Froebel and Pestallozzi, (Windschitl, 1999; Henson 2003). He emphasized learning in context. He explained that effort should be centred on the child rather than on the subject matter and that the most important effect education can have on a child is to instil in him/her the desire to go on learning (Windschitl, 1999; Henson 2003). Similarly, in 1919, Helen Parkhurst founded the Dalton School based on the principles that school programs should be adapted to the needs and interests of the students, should be learner centred, and that students should work to become autonomous learners (Semel, 1999). Similarly, the progressive education association (1919-1941) was based around a learner centred approach (Graham, 1967). In a study conducted from 1932-1940, it was found that this movement succeeded in enabling pupils to develop superior creativity, leadership skills, drive and objectivity (Windschitl, 1999). Unfortunately, however, public demand for a return to more traditional approaches to education resulted in it's demise. Following the success of the Russians in the space race of 1957 with the launch of Sputnik 1, Americans became distrustful of progressive education and demanded a return to basics in an effort to remain ahead in future similar situations (Matthews, 2000).

Constructivist education, although not a new epistemology, revolutionises the approach to teaching and learning, particularly in the sciences, and continues to test long held beliefs and approaches. It revolutionises approaches to teaching and learning because the construction of knowledge and the responsibility for coming to understand new phenomena is a shared enterprise between student and teacher. 'Learning is not the

passive acquisition of associations between stimuli and responses, but is rather the result of an active process of sense making on the part of the learner (Wiliam, 2003: 475).

2.4.2 Piagetian theory

Jean Piaget is described as 'by far the most influential theorist in the history of child development' (Shaffer and Kipp, 2007:243). Piaget (1896-1980) however, deserves credit for giving the impetus to the constructivist movement that has taken place during the twentieth century. Piaget devoted much of his life to the study of child development and the learning process. One of the basic premises upon which much of his work in the theory of constructivism is built, is that for learning to take place, the child's view of the world must come into conflict with his/her actual experience. Essentially, friction in the learner's current levels of understanding is central to Piagetian theory. When the child makes an effort to reconcile the two, his/her incomplete view of the world and his/her experiences of the world that learning will occur. Consequently, cognition develops through the refinement and transformation of mental structures or schemes (Piaget and Inhelder, 1958).

Piaget believed in two inborn intellectual processes: organisation and adaptation. Organisation is the process by which children combine existing structures into new and more complex intellectual schemes (Shaffer and Kipp, 2007). Adaptation is the process by which children adapt to their environment through assimilation and accommodation (Shaffer and Kipp, 2007). In the process of assimilation children try to interpret new experiences in terms of the existing schemes that they already possess and, in accommodating them, modify existing structures to account for new experiences. According to Piaget, disparities between one's internal mental schemes and the external environment stimulate cognitive activity and intellectual growth (Piaget and Inhelder, 1958). The assumption that underlines Piaget's view of intelligence as a basic life function that helps an organism adapt to its environment, is that if children are to know something they must construct the knowledge themselves. The child is described by Piaget as innately constructivist, an individual who plays with novel objects and gains an understanding of their essential features.

Piaget (1957) describes four stages of intellectual development which form a sequence that the child goes through from birth to adolescence. These stages are based on the idea that the developing child builds cognitive structures for understanding by responding to physical experiences in his/her environment. Briefly, they are:

Sensorimotor stage: At the sensorimotor stage of development, the rate of the development of the child is influenced by his/her cultural experience and the value placed on particular skills. Furthermore, the child develops through physical interaction with his/her environment with no knowledge of the concept of permanence of objects outside of his/her sight (Piaget, 1957).

Pre-operational/conceptual period: During the pre-operational/conceptual period children have an understanding of class membership, but they cannot differentiate between members of that same class. Piaget tells of his son who sees a snail and then after a few steps sees another snail and believes it to be the same snail. This is transductive thinking (Piaget, 1957). It is age appropriate; children think from object to object or from event to event. It is different to inductive thinking whereby one might use a number of facts in an effort to achieve an answer to a problem. Transductive thinking explains why many young children might ascribe life to any object that moves because it moves (Piaget and Inhelder, 1958). Children begin to think logically during this second stage of development. Number conservation however, is still difficult for children during this period. Although the child may distinguish between classes and members of classes, conservation of number may at this time trouble them. The major achievement of the child during the preoperational/conceptual stage of development is the development of language. This allows the child to develop at an increased pace because the linguistic symbol may, at times, replace the physical object in the cognitive process (Piaget, 1957). Egocentricity is a common feature of this stage of development; the child may only pay attention to a limited number of aspects of an object while observing it.

Concrete Operations: During the stage of concrete operations children can think more systematically and quantitatively (Piaget, 1957). Flavell, Miller and Miller (1993) describe operations as systems of internal mental actions that underlie logical thinking. Children may no longer be bound by centration or egocentrism, but may still require

opportunities to manipulate physical materials. At this stage, children can conserve number, area and liquid, and apply logical reasoning to problems that are presented to them. In number work for example, children can order objects in terms of dimension. Children develop the ability to order objects, known as seriation, during this stage. Brown (1970) explains that children are ready for formal education in mathematics depending on their classification and seriation abilities.

Formal Operational Stage: From the age of 12 and the child enters the period of formal operations. The child at this stage of formal operations can approach mathematical problems intellectually, in a systematic fashion (Piaget, 1957).

Formal thought reaches its fruition during adolescence. The adolescent, unlike the child, is an individual who thinks beyond the present and forms theories about everything, delighting especially in considerations of that which is not. The child, on the other hand, concerns himself only with action in progress and does not form theories (Piaget, 1957:148). This is the stage of abstract reasoning. It is reasonable to suggest that, at times, even adults do not operate at this stage. Brown (1970) suggests that other cultures do not value the stage of formal operations as much as our western culture does.

In conclusion, from a Piagetian perspective, learning is an internal process that occurs in the mind of the individual and cognitive conflict is essential to the learning process. Cognitive development, the productive of cognitive conflict, is an active process in which children are regularly seeking and assimilating new experiences, accommodating their cognitive structures to these experiences, and organising what they know into new and more complex schemes (Shaffer and Kipp, 2007). The classroom implications for education from a Piagetian perspective are clear; teachers must choose problems that are open-ended, i.e. that can be solved in many different ways, and all students must be engaged in the problem-solving process because it encourages, or indeed, forces them to think (Gredler, 2001).

2.4.3 Vygotskian theory

Lev Vygotsky has been credited with the development of social constructivist theory. Vygotsky was a Russian psychologist, and during his years at the Institute of Psychology in Moscow (1924–1934) he expanded his ideas on cognitive development. Vygotsky's writings emphasised the roles of historical, cultural and social factors in cognition and argued that language was the most important symbolic tool provided by society. Vygotsky's socio-cultural theory claims that cognitive growth is heavily influenced by culture and may be nowhere near as universal as Piaget assumed (Vygotsky, 1999). Vygotsky insisted that cognitive growth occurs in a sociocultural context which influences the form it takes, and that many of a child's most noteworthy cognitive skills evolve from social interactions with parents, teachers, and other more competent associates (Vygotsky, 1999). Children solve problems and interpret their surroundings in the context of the demands and values of their culture.

Vygotskian theory asserts that the intellect of the child is developed in the social environment.

> We propose that an essential feature of learning is that it creates the zone of proximal development; that is, learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. Once these processes are internalised, they become part of the child's independent developmental achievement (Vygotsky, 1978:90).

As discussed, Piaget believed that cognitive development consists of four main periods of cognitive growth: Sensorimotor, Preoperational, Concrete Operations and Formal Operations. In effect, Piaget saw an endpoint in the development of the intellect. Vygotsky, on the other hand, believed that cognitive development continued from birth right on up to death. His theory of development focuses on the significance of social interaction and culturally mediated tools such as language. Whereas an interpretation of Piaget can lead to the conclusion that teachers perform best when they get out of the way and let nature take its course, Vygotskian theory requires an involved teacher who is an active participant and a guide for the student. Vygotsky (1987:21) states, 'What

the child is able to do in collaboration today he will be able to do independently tomorrow'. Vygotsky emphasised the complexity of the development process and did not subdivide it into stages or categories. He proposed that cognitive development is profoundly influenced by social interaction. Vygotsky (1978) asserts:

> Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relationships between individuals (Vygotsky, 1978:57).

Vygotsky emphasised that social learning leads to cognitive development. The Zone of Proximal Development, according to Vygotsky is central to this socially mediated cognitive development. Vygotsky described it as 'the distance between the actual development level as determined by independent problem-solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers' (Vygotsky, 1978:86). In essence, the Zone of Proximal Development is where the child's unorganised concepts meet with and are extended by the logic of adult reasoning.

Vygotsky also recognized the importance of childhood play in the development of the child. He believed that play is the best preparation for adult life and is 'self' education and allows the child to enter the ZPD without the assistance of an adult (Vygotsky, 1999). Play helps the child develop the capacity to symbolise. Goals and rules become a focus of play as children enter school age, and play becomes an early mechanism for self-mastery: 'A child's greatest self-control occurs in play' (Vygotsky, 1978:99). While playing, the child uses his/her imagination and so begins to act independently. The child is moving from the situational constraints of early childhood towards the context free, abstract thinking of adulthood (Vygotsky, 1978).

Vygotsky reveals that language plays a central and powerful role in any learner's understanding of their cultural and historically embedded experience. Through language, meaning and sense are created. In Vygotsky's schema, language is far more important than it is in Piaget's (Vygotsky, 1962). Whereas Piaget believed that the

egocentric speech used by the child disappeared once the child moved on to the period of Formal Operations, Vygotsky viewed this as a transition from social speech to internalised thoughts (Driscoll, 1994). It is through conversations with adults that the child progresses and it is the innate need to communicate with and to understand that presses the child to seek meaning. Also, the child's own language comes to serve as his or her primary tool of intellectual adaptation (Driscoll, 1994). Vygotsky believed that the internalisation of tools such as language led to higher level thinking skills in developing intellects (Vygotsky, 1978).

Vygotsky's work focussed on the fundamental role played by social interaction in the development of cognition, the key difference between Piaget's and Vygotsky's theories of development. Vygotsky (1978) explained that the potential for cognitive development depends upon the Zone of Proximal Development and that, in turn, is dependent on full social interaction. According to Vygotsky (1987) while a child can perform some skills independently other skills cannot be performed even with help. Between these two extremes are skills that the child can perform with help from others. These skills are in the Zone of Proximal Development. Leontiev (1978) explains that the degree to which the child masters everyday concepts shows his actual level of development, and the degree to which he has acquired scientific concepts shows his Zone of Proximal Development. Cleborne, Johnson, and Willis (1997) explain that Vygotsky's Zone of Proximal Development emphasises his belief that learning is a socially mediated activity.

Daniels (1996) explains that the underlying assumption of the Zone of Proximal Development is that psychological development and instruction are socially imbedded, and that to understand them one must examine the surrounding society and its social relations. The type of teaching instruction therefore becomes pivotal to the process of development. Teachers must provide appropriate instruction, cognisant of the social environment, which challenges the children sufficiently to extend their level of understanding, but that also is correlated with current levels of understanding. The application of Vygotsky's theory of cognitive development in the educational setting is clear. Students must play an active role in their own education by collaborating with the teacher. Schaffer (1996:262) explains 'the adult does not impinge and shape an inert child but instead must act within the context of the child's characteristics and

ongoing activity. What ever effects are produced emerge from a joint enterprise to which the child as well as the adult contributes'. In collaboration, the adult provides motivation in the problem solving situation. The adult also provides requests and suggestions with respect to objects the child is working on or alternatively directing the child's attention before any action is demanded (Schaffer, 1996). Adult activities during collaboration are either supportive or challenging in nature. The former serves to maintain the current behaviour of the child and the latter serves to gear demands to the child's particular abilities at an appropriate pace for the individual (Schaffer, 1996). Specifically, Heckhausen (1987) suggests that the adult should focus on aspects of the task that lie beyond the level the child has obtained. Traditional schools have not promoted such collaboration and favour more teacher directed activity (Matthews, 2000). Vygotsky's theory implies the use of peer tutoring, collaboration, and small group instruction. Vygotsky's theory of intellectual development requires the teacher to organise learning at a level just above the current developmental level of the individual pupil. He explains that 'learning which is oriented toward developmental levels that have already been reached is ineffective from the viewpoint of the child's overall development' (Vygotsky, 1978:89).

> Teachers in Vygotsky's classroom would favour guided participations in which they structure the learning activity, provide helpful hints or instructions that are carefully tailored to the child's current abilities, and then monitor the learner's progress, gradually turning over more of the mental activity to their pupils (Shaffer and Kipp, 2007:281).

Both Piaget and Vygotsky stress the need for active rather than passive learning. The resultant implications for teachers, arising from their theories of cognitive development, are that they must take care to assess what the individual student already knows before estimating what he/she is capable of in the future. The next section involves a discussion of constructivism from various perspectives in an effort to identify how constructivist theory has evolved and how it arrived in our classrooms.

2.5 Constructivism: Radical, social or emergent?

The constructivist theory of learning has been introduced in the examination of both Piaget's and Vygotsky's work. Both Piaget and Vygotsky have been credited with sowing the seeds for the modern day constructivist movement. Three particular perspectives on constructivism will now be examined that have stemmed from this work, the radical, social and emergent perspectives. The latter draws on both the radical perspective which identifies learning as a series of cognitive reorganisations of the individual (Von Glasersfeld, 1995), and the social perspective which emphasises learning as a social accomplishment (Bauersfeld, 1992). The emergent perspective appeared during the early 1990s when American researchers came to a consensus that constructivist and sociocultural approaches were at least partially complementary (Cobb and Bauersfeld, 1995). Social constructivism is distinct from radical constructivism in that it insists that knowledge creation is socially mediated. In examining the work of von Glasersfeld (1992) on radical constructivism and Bauersfeld's (1992) studies on the importance of the social perspective on any knowledge or way of knowing, one comes to realise that knowledge exists in the mind of the learner only because the learner is part of a broad sociocultural setting.

> It is useful to see mathematics as both cognitive activity constrained by social and cultural processes, and as a social and cultural phenomenon that is constituted by a community of actively cognising individuals (Cobb et al., 1992: 3).

The following table gives a broad outline of the three perspectives of constructivism that are the subject of discussion throughout the next section of the literature review. This table indicates the origins of the three perspectives, the assumptions about learning from each perspective, the implications for teaching from each of the perspectives and finally, the implications for the mathematical problem solving classroom.

Table 1: Constructivist perspectives and the implications for the classroom

Perspective	Assumptions about Learning	Implications for Teaching	Implications for the Mathematical Problem Solving Classroom	Nature of Knowledge
Radical Perspective	Knowledge construction occurs as a result of the individual working through dilemmas.	Hands on activities, discovery learning, working with manipulatives, questioning techniques that are specifically designed to probe the beliefs of the child.	The individual child engages in solving mathematical problems utilising materials available to him/her in his environment.	No such thing as knowledge independent of the knower, but only knowledge constructed by the individual as he/she learns. It is not a transferable commodity.
Social Perspective	Knowledge construction occurs from social interactions within which cultural meanings are shared by the group and then internalized by the individual.	Co-operative learning, collaborative learning situations, the lived experiences of the students integrated into classroom co-operative situations	Mathematical problems are tied to the students' activity, context and culture.	The surrounding context of the learner in interactions with others will determine viable knowledge.
Emergent perspective	The construction of knowledge has both individual and social components and these cannot be separated in any meaningful way.	Students work collaboratively and are supported as they engage in task oriented dialogue.	Students solve problems, explain their ideas, utilise appropriate materials and manipulatives and reflect upon the experience in collaboration.	Knowledge is both an individual and a social construction. In the creation of knowledge, individual and social domains complement each other

In the next section, each perspective will be examined in detail.

2.5.1 Constructivism: Radical perspective

> It makes no sense to assume that any powerful cognitive satisfaction springs from being told that one has done something right, as long as 'rightness' is assessed by someone else. To become a source of real satisfaction, rightness must be seen as the fit with an order one has established oneself (von Glasersfeld, 1987:329).

The radical constructivist regards the purpose of education as educating the individual child in a fashion that supports the child's interests and needs; the child is the subject of study and individual cognitive development is the emphasis (Vadeboncoeur, 1997). Knowledge construction occurs as a result of working through dilemmas provided by the teacher. Therefore, classrooms that are organised from a radical perspective will involve hands on activities, discovery learning, the use of manipulatives and tasks designed to challenge existing concepts and thinking processes, coupled with questioning techniques which specifically test and probe the beliefs of the individual child. The difficulty with constructivism from this perspective is its lack of attention to the influence of classroom culture and the broader social context where the child resides (Vadeboncoeur, 1997).

Radical constructivism is just that, radical; no knowledge can claim uniqueness. In other words, no matter how viable and satisfactory the solution to a problem might seem, it can never be regarded as the only possible solution (von Glasersfeld, 1996). In the classrooms of today, it is difficult to comprehend how we might educate from a radical perspective. The success of teaching from this perspective depends on minimal interruption of cognitive growth by environmental factors such as the teacher. Some key elements of radical constructivism can be seen in curricula such as the use of concrete manipulatives, discovery learning methods and hands on activities. In particular, radical constructivists emphasise the importance of concrete objects in the creation of knowledge especially amongst children of a young age (von Glasersfeld, 1987). Listening becomes the key to successful teaching following a radical framework. The teacher must listen to the explanations of, for example, the counting efforts

of a young child, pick out fragments of knowledge, and develop situations to exploit these fragments. Teachers must devise situations to test the pupils' constructions in an effort to help them reorganise their understanding.

Radical constructivists believe in the reorganisations of mental schema by students, each more encompassing and integrative than its predecessor (Prawat and Floden, 1994). Getting stuck in any given situation is, therefore seen as a weakness in the learning environment. This affects the opportunity for cognitive growth, and is therefore a challenge for the teacher. Radical constructivists agree that an individual must reconcile problems in his/her understanding by attempting to associate them with his/her experience. Radical constructivism ignores any direct social element of learning which is a critical difficulty given the structure of schools. It is not that the radical constructivist ignores the existence of the social context of the learner but believes that the learner will determine the usefulness of the social context or any part of it if the learner deems it suitable for their learning experience. In other words, as the experiential world of the learner involves others, radical constructivists argue that other learners must have significant influence on individual experiences. However, it is up to the learner to perceive the nature of any experience and determine the extent of its usefulness.

Both the cognitive activity and the experiential world of the individual remain central to the radical constructivist's theory of knowledge. The individual must assume control over his learning and achievements. According to von Glasersfeld (1996:340),

> It is the knower who segments the manifold of experience into raw elementary particles, combines these to form viable 'things', abstracts concepts from them, relates them by means of conceptual relations, thus constructs relatively stable experiential reality.

Radical constructivists perceive replacing any misconception with any conception considered correct by another individual as unhelpful to the learner (Von Glasersfeld, 1989). From a constructivist teaching perspective, the

dismissal of a child's efforts as wrong demolishes student motivation, and, furthermore, replacing any student's struggles with an exemplar of the right way is counterproductive (Windschitl, 1999). From a radical constructivist's perspective there is no place for a teacher that assumes the role of knowledge transmitter. One can question whether or not radical constructivism can ever lead to the development of a community of knowledge or, indeed, any knowledge discipline. From a radical perspective, all constructions can be valid. However, one of the difficulties with radical constructivism is that these constructions may contradict established knowledge and practice, knowledge and practices that have existed and evolved over centuries.

2.5.2 Constructivism: Sociocultural perspective

Social constructivism reflects a theory of human development which situates the development of the student within a sociocultural context. Individual development is derived from social interactions within which cultural meanings are shared by the group and then internalized by the individual (Richardson, 1997). Individuals construct knowledge by engaging with the environment and in the process they are changed. From the late 1960s or early 1970s, social constructivism became a term applied to the work of sociologists of knowledge including Barnes, Bloor, Knorr-Cetina, Latour, and Restivo (Windschitl, 1999). What they share, is the notion that the social domain impacts on the developing individual in some crucially formative way, and that the individual constructs meanings in response to experiences in social contexts.

The development of the sociocultural perspective on constructivism can largely be attributed to Lev Vygotsky. Co-operative learning is central to Vygotsky's socio-cultural perspective. Vygotsky (1978) explained that, through co-operative group learning, pupils were encouraged to enable fellow pupils to understand before any pupil was awarded marks. Vygotsky championed a positive, active approach to education. Central to Vygotsky's sociocultural approach is the claim that higher mental functions in the individual have their origins in social life, but also that an essential key to understanding human, social and psychological processes is the use of tools and signs used to mediate them (Wertsch, 1990).

Such tools and signs are critical in knowledge development and acquisition, and are intrinsic to the socio-constructivist perspective. They are unique to human existence, and are crucial in linking human beings with the environment and the past.

Similar to Vygotsky's socio-cultural perspective, contemporary theorists such as Lave (1988) and Rogoff (1990) propose situated cognition. Situated cognition presumes that cognitive activity is so context bound that one can never distinguish between the individual's cognitive ability, the individual's affective state, the context in which activity takes place, and the activity itself.

> Viewing the world of a persons ideas, beliefs and knowledge as autonomous – essentially disconnected from their bodily (i.e. lived) experience, and hence from their sociocultural context – provides broadly for a devaluing of lived experience in favour of 'higher' abstracted contemplative activity (Kirschner and Whitson, 1997:4).

Situated cognition is a theory of instruction which suggests that learning is naturally tied to authentic activity, context, and culture (Brown, Collins, and Duguid, 1989). It embodies a socio-constructivist perspective of education and is particularly relevant to mathematics education. In situated approaches students collaborate with one another and their instructor in achieving some shared understanding. The situated cognition perspective on the development of students' mathematical abilities advocates the employment of informal sociocultural settings in classroom situations, as they can have a positive impact on students' problem solving-abilities. Exploring conventional arithmetic with students in the school environment does not explicitly assure that subsequent knowledge can be utilised in real life situations, since traditional approaches to problem-solving have focussed the student on the written symbol. Therefore, students tend to lose track of the transaction they are quantifying. Carraher and Schliemann (1985) studied youngsters aged 9-15, with various amounts of schooling experience, solving mathematical problems both in the school and everyday settings. They found that youngsters' performances in the natural setting, in cooperation with others, were significantly better than those situated in

the classroom environment. Moreover, they also found that problems provided in the school setting were more likely to be solved through the employment of school algorithms, whilst those presented in the natural setting were solved utilising oral procedures (Carraher and Schliemann, 1985).

2.5.3 Constructivism: The emergent perspective

> Learning is both an act of individual interpretation and negotiation with others. Knowledge in the various disciplines, then, is a corpus of constructions that are subject to change as different kinds of evidence are discovered and members of disciplinary communities debate about new ideas becoming part of the canon (Windschitl, 1999: 34).

The social perspective is an interactionist view of communal or collective classroom processes. The psychological or radical perspective is a psychological constructivist's view of individual student's activity as they participate in and contribute to the development of communal processes. 'The coordination of interactionism and psychological constructivism is the defining characteristic of the version of social constructivism that is referred to as the emergent perspective' (Cobb and Bauersfeld, 1995: 176). This emergent perspective emphasises the social processes and views knowledge as having both individual and social components and hold that these cannot be viewed as separate in any meaningful way (Cobb and Bauersfeld, 1995). The difference between the two previous perspectives lies in the fact that social constructivists see learning as increasing one's ability to participate with others in meaningful activity and radical constructivists focus on how individuals create more sophisticated mental representations using, manipulatives, information and other resources (Wilson, 1996). The emergent perspective is a synthesis of radical and social perspectives which claims that knowledge is personally constructed and socially mediated (Tobin and Tippins, 1993).

According to the social constructivist, the constructive processes are subjective and developed in the context of social interaction. Experimenting with or modifying our perception of the environment is an extension of a realist illusion (von Glasersfeld, 1987). Students arrive at what they know about mathematics

through participating in the social practice of the classroom, rather than through discovering external structures which exist independent of them. Cobb and Yackel (1996) and Stephan and Cobb (2003) describe the emergent perspective as a version of social constructivism. This theory draws from constructivist theories which identify learning as a series of cognitive reorganisations of the individual (von Glasersfeld, 1995) and interactionist theories which emphasise learning as a social accomplishment (Bauersfeld, 1992). The emergent perspective attempts to reconcile radical and social constructivism. Cobb and Yackel (1996:37) explain that students reorganise their learning 'as they both participate in, and contribute to, the social and mathematical context of which they are part'.

Murray (1992) and Cobb and Yackel (1996) argue that mathematical knowledge is both an individual and a social construction and that individual and social dimensions of learning complement each other.

> The two key features of the account are as follows. First of all, there is the active construction of knowledge, typically concepts and hypotheses, on the basis of experience and previous knowledge. These provide a basis for understanding and serve the purpose of guiding future actions. Secondly, there is the essential role played by experience and interaction with the physical and social worlds, in both the physical action and speech modes. This experience constitutes the intended use of the knowledge, but it provides the conflicts between intended and perceived outcomes which lead to the restructuring of knowledge, to improve its fit with experience (Ernest, 1991:72).

Bauersfeld (1988) and Voigt (1992) have elaborated on the relevance of the emergent perspective for mathematics education research. Both cultural and social processes are integral to mathematical activity (Voigt, 1992). Mathematical learning opportunities arise when children attempt to make sense of explanations given by others and when they compare others' solutions to their own (Cobb and Yackel, 1996). From the emergent perspective, doing maths is a social activity as well as an individual activity. In the negotiation of norms within the classroom, the teacher has the central role of initiating and guiding the

formation of these norms, but the individual student has an active role in this formation as well (Cobb and Yackel, 1996).

Windschitl (1999) has derived the key features of constructivist classrooms from this 'hybrid' (Windschtl, 1999:137) view of constructivism. Critically, they connect what is known about how people learn and the classroom conditions that optimize opportunities to learn in meaningful ways. These conditions can be cross-referenced with current literature (Schoenfeld, 1992) to illustrate appropriate teaching of mathematical problem solving.

- Teachers elicit students' ideas and experiences in relation to key topics, then fashion learning situations that help students elaborate on or restructure their current knowledge.
- Students are given frequent opportunities to engage in complex meaningful, problem based activities.
- Teachers provide students with a variety of information resources as well as the tools (technological and conceptual) necessary to mediate learning.
- Students work collaboratively and are given support to engage in task-oriented dialogue with one another.
- Teachers make their own thinking processes explicit to learners and encourage students to do the same through dialogue, writing, drawings or other representations.
- Students are routinely asked to apply knowledge in diverse and authentic contexts, to explain ideas, interpret texts, predict phenomena, and construct arguments based on evidence, rather than to focus exclusively on the acquisition of pre-determined right answers.
- Teachers encourage students' reflective and autonomous thinking in conjunction with the conditions listed above.
- Teachers employ a variety of assessment strategies to understand how students' ideas are evolving and to give feedback on the processes as well as the products of their thinking.

In summary, teachers must engage in listening exercises to identify a student's ideas and experiences so that in turn he/she can devise appropriate learning situations and make available appropriate tools and resources which might be required for use by the students. Students should engage with one another in problem solving situations; designing, testing, debating and reflecting upon situations to reach appropriate conclusions. Particular emphasis is placed on extension activities that arise out of constructivist learning situations to validate, extend, refine and predict the usefulness of the learning exercise in future situations. Now that constructivism has been explored from key perspectives, it is essential to consider the associated teaching implications for mathematical problem solving. Before this however, it is important to understand the difficulties teachers have in relation to the teaching of mathematical problem solving.

2.6 Difficulties in relation to the teaching of mathematical problem solving

Much of the instruction in basic mathematical skills can be characterised as having a singular focus on the development of skill automaticity through extended practice on daily assignments of computation problem sets (Schoenfeld, 2004). In contrast, the stated purposes of many problem-solving curricula are on the development of higher order skills and the development of cognitive flexibility. These curricula provide a different and difficult set of pedagogical concerns for teachers planning problem-solving instruction. However, Burns and Lash (1988) found that the majority of teachers focus more on showing students how to do mathematics than on getting students to understand something new on their own. Teaching problem-solving can cause difficulties for the teacher.

Carpenter, Fennema, Peterson, Chiang and Loef (1989) emphasise the importance of posing problems to students and listening to how students describe the way they have solved such problems. However, Fosnot (1989) explains that teachers are unfamiliar with such teaching because they are products of a system that emphasised drill and procedure. Fosnot (1989) suggests that teachers enjoy the safety provided by workbook pages, computation sheets, and drill during

instruction, often because they themselves are products of a similar approach. Children must be enabled to interpret and develop an understanding of mathematical processes rather than simply just the ability to perform these mathematical processes. The ultimate responsibility for this resides with the teacher. This said, Ball (1996) contends that teacher confidence impedes the exploration of problem-solving in this manner because,

> When we ask students to voice their ideas in a problem-solving context, we run the risk of discovering what they do and do not know. These discoveries can be unsettling when students reveal that they know far less than the teacher expected and far more than the teacher is prepared to deal with Ball (1996).

In relation to classroom resources, Bauersfeld (1995) highlights the poverty and restrictedness of mathematical visualisations presented to children in textbooks. Textbooks are not sufficient as a source of challenging mathematical problems that will provide the impetus for debate and discussion in the mathematics class. Opportunity for real engagement in problem solving will come from the teacher designing and informing particular mathematical problems that will suit the needs of the students involved. Ng (2002) found that textbooks provide a high portion of routine, closed-ended problems and problems with exactly sufficient information. This is also consistent with the Irish mathematical textbook (Harbison, 2009). Ng (2002) also found that examples modelled placed no emphasis on the final stage of Polya's (1945) problem-solving model which is crucial to the problem solving process from a constructivist perspective as it is the time for debate, discussion conjecture and analysis. Heuristics suggested by curricula are not all covered by the traditional textbook, and textbooks need to include more open-ended problems, non-routine problems, authentic problems and problems with insufficient or extraneous information as well as other traditional problems (Fan and Zhu, 2007). Otherwise, teachers must deviate from textbooks and utilise their own resources. Textbook problems are solved using traditional school mathematics and allow little opportunity for the development of alternative strategies of solution. The following examples taken from an Irish primary mathematics textbook illustrate this

- The diameter of a coin is 1.27 centimetres. What is the total length of 9 such coins (O'Loughlin, 2003: 186)

- Paul wrote the number 6.87. Susan wrote a number 9 times that amount. What number did Susan write? (O'Loughlin, 2003: 78).

Roth and McGinn (1997) reveal that school problems posed by textbooks require algorithmic approaches to achieve a solution. In school like problems, the answers are already implied although withheld or concealed by the problem statement. Roth and McGinn (1997:19) state: 'There is virtually no carry over to everyday problem solving: there exists a chasm between the problem-solving practices one needs to be successful in schools versus those needed in everyday life'. The answer is pre-figured in advance so that actual solution paths can be assessed against the ideal solution. Teachers must have the knowledge and dispositions of effective problem solvers to move from traditional beliefs about mathematical problem-solving and textbooks (Roth and McGinn, 1997). To accomplish this, teachers themselves need to become problem solvers and realise what might constitute an appropriate mathematical problem for use with their students.

Research shows that elementary school students' lack of success in and fear of problem-solving stems from the negative attitudes and incompetence harboured by their teachers (Schoenfeld, 1992). Insecurities can be traced back to a teacher's own experience of mathematics, which then leads to rigid and stereotyped curricula and methods and a heavy reliance on texts (Wilburne, 2006). To counteract this, Wilburne (2006) argues that non-routine mathematical problems should be assigned during elementary mathematics content courses. Non-routine problems pose rich and meaningful mathematical experiences. Often they have no obvious solution. These problems pose questions that require students to use various strategies and that spark student's interest in engaging in the problem solving process' (Wilburne, 2006: 454). Non-routine problems can be used to encourage logical thinking, reinforce or extend pupils' understanding of concepts, and to develop problem-solving strategies which can be applied to other situations. The following is an example of a non-routine problem:

What is my mystery number?

If I divide it by 3 the remainder is 1.

If I divide it by 4 the remainder is 2.

If I divide it by 5 the remainder is 3.

If I divide it by 6 the remainder is 4.

By posing such problems, teachers become more positively disposed to the problem-solving process and by having experience in the process of solving problems they in turn are more aware of how to facilitate primary students. However, fear on the teacher's part is often an inhibiting factor in the utilisation of these problems in the classroom (Wilburne, 2006).

O'Shea (2003) investigated the teaching of mathematical problem-solving amongst Irish primary mathematics teachers at senior class level in the primary school. In analysing the data, O'Shea (2003) found that 62 per cent of the survey population used all problem-solving lessons to facilitate the practice of number concepts and skills. Teachers very often use methods of teaching that mirror those methods that were used to teach them (Lortie, 1975). Ball (1996) and Taback (1992) have found that the majority of teachers have not experienced mathematics as a discipline involving problem solving. They have rather experienced mathematics as a fixed body of knowledge to be learned. This creates a problem as the Primary Mathematics Curriculum (Government of Ireland 1999a; 1999b) recommends that problem-solving activity should play an integral role in a student's mathematical learning. Furthermore, O'Shea (2003) found that 82 per cent of Irish teachers indicated that they chose not to study either the subject mathematics or the teaching of mathematics to any significant extent at third level. It appears that the mathematical experience of the teacher is an obstacle to the use of mathematical problem solving in teaching.

Teachers must have a thorough knowledge of their domain and must organise instruction at an appropriate level for each student (Carpenter and Fennema, 1991). To ensure teachers are equipped with the necessary skills to explore mathematics from a problem-solving perspective, the appropriate and rich professional development opportunities must be made available to them. Shiel

and Kelly (2001) have established that a significant number of pupils are taught by teachers who have not engaged in professional development related to the teaching of mathematics in recent years and, where teachers have engaged in such professional development, they have deemed the quality of such courses uneven (Shiel and Kelly, 2001). Teachers have engaged with in-service education provided by the Department of Education and Science since the introduction of the Primary School Curriculum in 1999. O'Shea (2003) revealed low satisfaction levels with in-service education related to investigating mathematical problem-solving with children at 5th and 6th class levels. Specifically, survey respondents indicated mathematical problem-solving required a more in-depth focus on utilising group activity. Indeed, Shiel and Kelly (2001) highlight that the matter of grouping pupils for mathematics, and for exploring problem-solving from a constructivist perspective as espoused by the curriculum, continues to challenge schools and teachers. O'Shea (2003) found that 25.4 per cent of respondents almost never allow students to work in small groups/pairs during problem solving lessons without any significant teacher influence. In other words, teachers in the Irish primary school have significant control of the actions of pupils as they engage in mathematical problem solving leaving little opportunity for student debate, student trial and error and the construction by the student of different solution strategies. The mathematics curriculum recommends strongly that students should operate in pairs or small groups to solve problems co-operatively. O'Shea (2003) found that 57.5 per cent of survey respondents reported that students work together as a whole class with the teacher instructing the class during most lessons.

Constructivist theory has the power to refocus education on progressivism, even though coordinating learning from a constructivist perspective is challenging. Only through significant professional development and with sustained support and assistance will effective change occur (Day, 1999). The most profound challenges for teachers are not merely associated with the acquisition of new skills but with making personal sense of constructivism as a basis for instruction, reorienting the cultures of classrooms to take account of constructivist philosophy, and dealing with the pervasive educational conservatism that works against efforts to teach for understanding (Apple, 1982; Little, 1993; Purple and

Shapiro, 1995). There are a number of factors that impact on the use of constructivist philosophy by teachers. For example, when education is driven by a significant focus on results and achievement and on educational conservatism it can be difficult to implement a constructivist philosophy. Traditional methods of instruction have delivered these results and educational systems are slow to change. As a result of this Pirie and Kieran (1992) have claimed that teachers have distorted the original notion of constructivism because they wanted to be perceived as doing the right thing. Christiansen (1999) has explained that teachers need to think about teaching, and ideas about teaching, as there are no limits to the potential for development. However, the teacher can hinder any change of direction if it is seen as a threat to his/her professionalism and efficacy. This is particularly true when a change is seen as dramatic. Furthermore, the traditional approach to mathematics instruction enables teachers to build a sense of self-efficacy by defining a manageable mathematical content that they have studied extensively and by adopting clear prescriptions for what they must do with that content to affect student learning (Draper, 2002). The traditional role of the teacher must be deconstructed so as to take account the qualities and dispositions of the learner.

The overarching difficulty in engaging children in learning from an emergent constructivist perspective in our classrooms, lies in the fact that effective forms of constructivist teaching depend on nothing less than the reculturing of the classroom (Fullan 1993; Joseph, Bravmann, Windschitl, Mikel and Green, 2000). Reculturing is the process of developing professional development communities and includes attention to assessment, pedagogy and the development of norms that support improved teaching (Teitel, 2003). The features that make constructivist classrooms effective complicate the lives of teachers, students, administrators and parents (Windschitl, 2002). However, from a mathematical problem solving perspective, reorganising the cultures of classrooms to reflect constructivist principles will crucially ensure the engagement of students in, what Schoenfeld (1994) has described as, making sense of mathematics. If such reculturing is achieved then students will be engaged in problem solving as art (Stanic and Kilpatrick, 1988) because by its very nature, learning from a constructivist perspective implies conjecture, analysis, engagement, debate and

reflection which are among the essential skills central to becoming an efficient mathematical problem solver. The next section examines the teaching of mathematical problem solving from a constructivist perspective and places particular emphasis on strategies for introducing students to learning from a constructivist perspective in its presentation and discussion of problem solving heuristics.

2.7 Constructivism and teaching

There have been some critics of constructivism (Matthews 1993, Osborne 1996), and others urge caution in its adoption (Millar 1989) yet few would dispute Fensham's (1992) claim that 'the most conspicuous psychological influence on curriculum thinking in science since 1980 has been the constructivist view of learning' (Fensham 1992: 801). Pepin (1998) states that, 'the constructivist point of view makes it possible to develop a vision of the whole educational phenomena which is comprehensive and penetrating' (Pepin 1998:173). In both the Irish and American curricula, constructivism from an emergent perspective plays a central role.

Before examining the implications of the constructivist theory of learning for teaching, let us consider why teachers might employ constructivism in their classrooms. Hardy and Taylor (1997) explain that it offers teachers 'a moral imperative for deconstructing traditional objectivist conceptions of the nature of science, mathematics and knowledge, and for reconstructing their personal epistemologies, teaching practices and educative relationships with students' (Hardy and Taylor 1997:148). Constructivism, as a theory of learning, is attractive as activities and experiences associated with it are very much student oriented. When learning from an emergent constructivist perspective, students are engaged in hands on activities, collaboration with their peers and are building upon prior knowledge which is itself a product of similar construction. Also, the current primary curriculum suggests teachers employ constructivism from the emergent perspective. The mathematics curriculum (Government of Ireland, 1999a: 3) states 'to learn mathematics children must construct their own internal

structures… it is in the interpersonal domain that children can test the ideas they have constructed and modify them as a result of this interaction'. But, all this aside, deep rooted problems arise when attempts are made to apply constructivism from an emergent perspective within the classroom. Why, because it breaks the mould radically from traditional educational models in which teachers were schooled thus making it difficult to visualise constructivist pedagogy (Cobb, Yackel and Wood, 1991). Furthermore, Prawat and Floden (1994) explains that educators often place an inordinate amount of faith in the ability of students to structure their own learning, which is central to constructivist theory. Beyond engaging children in structuring their own learning, there are difficulties in relation to what happens following the learning experience. Specifically, these difficulties are utilising other problem solving situations that may arise (Elmore, Peterson and McCarthey, 1996). From a mathematical problem solving perspective, it is the lack of attention paid to conversations and problem solving opportunities that might arise following the solving of the particular mathematics problem. So although a particular problem solving exercise may prove worthwhile and children make appropriate sense of a mathematical problem, it is the extension of the mathematics that arises during the activity the teacher may fail to capitalise on. Bauersfeld (1995) explains that the adoption of constructivist principles in the theoretical modelling of learning and teaching processes in mathematics will lead to a radical shift of the meaning of many key concepts used as descriptions for classroom realities. For the mathematics classroom, emergent constructivism is a significant deviation from traditional conceptions of mathematics teaching (Cobb and Yackel, 1996). Constructivists urge discourse within the classroom which places significant responsibility on both teacher and student.

Teaching from a constructivist perspective is complex. However, there is compelling argument in favour of it. A behaviourist's approach to teaching involves didactic teaching strategies, and students, subsequently, can have difficulties with understanding. From a constructivist perspective, training is not enough; the key to understanding is in actively building upon prior experiences (von Glasersfeld, 1989). This 'understanding' is the building up of a

conceptual structure that is compatible in the mind of the individual. 'Many who are involved in educational activities continue to act as though it were reasonable to believe that the verbal reiteration of facts and principles must eventually generate the desired understanding on the part of the students' (Matthews, 1993:11). It is a little naïve to expect that repetition and rote learning can lead to understanding. Duckworth (1987) explained that 'meaning is not given to us in our encounters, but it is given by us, constructed by us, each in our own way, according to how our understanding is organized' (Duckworth, 1987:112). It is clear that a constructivist approach to education is critical to engaging students with their own learning, and, more importantly, ensuring these learning experiences are valuable, useful and meaningful. Why then are teachers prone to using a didactic approach to teaching and learning which can have little significant effect on the vast majority of students? Perhaps it is because reforms that that seek to change fundamental structures, cultures and pedagogies are difficult to sustain and progress (Cuban, 1988). Although the constructivist theory of learning may be attractive, this attractiveness is not enough in terms of re-orienting teaching practices that reflect a constructivist approach to learning. The reform of structures, cultures and pedagogies require sustained efforts at support and professional development (Day, 1999).

The drill and practice approach to mathematics instruction has an affinity for a static and timeless conception of mathematical truth (Schifter, 1996). Constructivism proposes a radical shift in this conception. Emergent constructivism requires active learning and involvement on the part of the pupil which allows for the creation of mathematical cultures (Schoenfeld, 1994). The development of critical thinking, the focus on authentic learning, and the introduction of a child centred curriculum puts the acquisition of knowledge squarely on the shoulders of the students (Resnick, 1987) and sits well with the emergent perspective on constructivism. However, teaching from such a perspective presents challenges. Representing content, respecting students, and creating and using community are not aims easily resolved (Ball, 1993). Knowing how to phrase things, to highlight certain aspects of any representation, while downplaying anything that can cause misconceptions, can be difficult (Ball, 1993). One of the most vexing issues faced by a constructivist can be

striking a balance between honouring the effort of the individual whilst managing to steer the group effort towards intellectually acceptable knowledge (Prawat and Floden, 1994).

From a mathematics perspective it may be prudent to adopt a literacy approach in mathematics education, as particular approaches used in the teaching of literacy are constructivist. Gallimore and Tharp (1989) suggest enabling mathematical students to read, write, speak, compute, reason, and manipulate both verbal and visual mathematical symbols and concepts. Indeed the Curriculum and Standards for School Mathematics (NCTM, 2000) recommend such an approach.

> Students who have opportunities, encouragement, support for speaking, reading, writing and listening in mathematical classes reap dual benefits: they communicate to learn mathematics and they learn to communicate mathematics (NCTM, 2000:60).

Also, according to Draper (2002), literacy activities designed to help students negotiate and create text, can be adapted for use in the mathematics classrooms. 'The end benefit of these adaptations can be a mathematics classroom that is responsive to the needs of all its students and falls in line with constructivist tenets of teaching and learning, engaging students and teachers in conversation around mathematical texts in a way that lets students negotiate and create their own texts' (Draper, 2002:531).

The teacher must assist the student in the development of acceptable and viable knowledge (Tobin and Tippins, 1993; Wheatley, 1991). Therefore, the teacher must join the fray and become an active participant and guide in the pursuit of mathematical knowledge. This implies that the teacher must question, infer, design, predict and facilitate to support the increasingly autonomous intellectual work of students. It is imperative teachers make the transition to a constructivist perspective because two features of traditional school mathematics give rise to difficulties and unnecessary stress. One is the premature move to the use of abstract numbers and the other is premature training in the symbols and conventional displays of arithmetical computations in isolation from meaningful situations involving numeracy (Hughes, 1986; Labinowicz, 1985). The

constructivist strategy would be to allow children use methods they are comfortable with. The constructivist teacher is less likely to force children to use mathematical methods or notation until they are comfortable with them, and know what they are doing. When children are engaged in an investigation the teacher organising learning from a constructivist perspective will respect the child's own efforts, and will try to avoid giving surreptitious assistance.

Mathematical problem solving constructivist classrooms are interactive, complex and unpredictable. They are highly charged environments involving considerable debate, discussion and argumentation (Windschitl, 1999). These classrooms are more difficult to manage than traditional classrooms because of the unpredictability of students' constructions and mathematical interpretations. The teacher is less likely to be able to confidently plan ahead unlike within traditional mathematical classrooms. Furthermore, the creation of a constructivist classroom is a significant task for a teacher as it involves much more than textbook chapters and seatwork. The rewards however are great. It is imperative therefore that teachers experience mathematical lessons that challenge them at their own levels of ability so that they can increase their knowledge and experience, a depth of learning that, for many, will be unprecedented. The teacher must become the problem solver and be provided with learning experiences that will challenge years of traditional forms of education. They must be invited to invent, extend, test, and debate rather than just do.

2.8 Constructivism and mathematical problem solving

Stanic and Kilpatrick (1988) distinguish three traditionally different views of problem solving. In one, problem solving is an act of solving problems as a means to facilitate the achievement of other goals such as teaching math. In another, problem solving is a goal in itself of the instructional process; it is a skill worth teaching in its own right. Finally, problem solving involving challenging problems can be viewed as a form of art, which for mathematicians is what math is ultimately about. Therefore, if we want children to engage in what mathematics is ultimately about, they must be engaged in the work of mathematicians which is mathematical problem solving. Current empirical

research suggests mathematical problem solving involves children working under conditions involving minimal intervention as they explore patterns, make conjectures test hypotheses justify solutions and engage collaboratively with one another (Francisco and Maker, 2005; Hoffman and Spatariu, 2007). This is a constructivist approach to learning.

Mathematical problem-solving from the emergent perspective on constructivism is central to the Primary School Curriculum (Government of Ireland, 1999a; 1999b). It encourages teachers to present children with 'real problems related to their own experience encouraging them to develop strategies for solving them imaginatively' (Government of Ireland, 1999a:47). Current literature urges the use of problem-solving activity to connect different ideas and procedures in relation to different mathematical topics and other content areas. It is through mathematical problem-solving that students attain mathematical power (Carpenter and Lehrer, 1999). Although a prerequisite for success in problem-solving is the acquisition of basic skills, the curriculum emphasises the use of constructivist practices so that the child will construct new mathematical knowledge as they solve mathematical problems.

Mathematical enquiry is based upon asking questions, following particular lines of inquiry, and having fun. The introduction of mathematical enquiry and investigation into the classroom provides a fresh perspective for both student and teacher (Jaworski, 1996). Very significantly, according to Ernest (1991: 283), 'the mathematical activity of all learners of mathematics, provided it's productive, involving problem posing and solving is qualitatively no different from the activity of professional mathematicians'. Surely this is the goal of mathematics education: that children become skilled mathematicians and that therefore employing an emergent constructivist approach to learning mathematical problem solving is essential?

Mathematical investigations initially became popular because of their introduction of the fun aspect into mathematics lessons (Jaworski, 1996). Overtime, they also promoted the development of mathematical processes that could then be applied in other mathematical work. Students became more

involved in mathematical deliberations and this was especially noticeable in classrooms that adopted Polya's ideas of having students guess and test out mathematical ideas. Specialising, generalising, conjecturing, and convincing became fundamental to mathematical classrooms. Furthermore, more emphasis began to be placed on process and method rather than on results, something that had been a focus of traditional mathematical classrooms. The National Curriculum, when introduced in the UK in 1989, emphasised the processes of doing mathematics (Jaworski, 1996). The Irish mathematics curriculum (Government of Ireland, 1999a; Government of Ireland, 1999b) and the United States' Principles and Standards for School Mathematics (NCTM, 2000) for students from kindergarten to grade 12 also reflect such a philosophy. Application, mathematical communication, reasoning, and logic, and proof have become integral to a student's mathematical explorations (Trafton and Midgett, 2001). This is a constructivist approach to mathematics education and it helps students to build actively upon current levels of understanding, thus ensuring that mathematics does not become abstract for the student.

It is useful to examine English and American reform initiatives in relation to the teaching of problem-solving since their curricula are closely related to the Irish approach to mathematics education. Investigative mathematics became a valued activity for the mathematics classroom following the publication of the Cockroft Report in 1982 in Britain (Cockroft, 1982). 'The idea of investigation is fundamental both to the study of mathematics itself and also to an understanding of the ways in which mathematics can be used to extend knowledge and to solve problems in many fields' (Cockroft, 1982: 50). This report (Cockroft, 1982) was especially interesting as it highlighted a significant difficulty regarding the teaching of problem-solving. The report explained that the willingness of the teacher to follow any line of inquiry the student may choose was integral to any investigation. Despite the recommendations of the report, however, investigations became isolated units of work, merely add-ons to the existing syllabus (Jaworski, 1996). The curriculum at the time prevented the teacher from having these mathematical investigations as core elements of their programmes because of the range of material that was presented to be explored with pupils and the various targets that had to be met. However, as mathematical

investigations became part of examinations at the General Certificate of Education Level (GCSE), they came to be used or taught (Jaworski, 1996). Many individualised schemes were established so that children could work at a pace suited to them, such as the Kent Mathematics Project (Kent County Council, 1995) and The Secondary Mathematics Individualised Learning experiment (SMILE, 1990). Unfortunately, when the need for assessment arose, and as examinations became stereotyped, practicable sets of procedures became prominent (Jaworski, 1996).

The Principles and Standards for School based Mathematics (NCTM, 2000) in the United States places less emphasis on the rote memorisation of isolated skills and facts in favour of emphasising communication within the mathematics classroom while engaging in problem-solving. According to Cobb, Perlwitz and Underwood (1998) such a change of direction, from a didactic pedagogy or a model of transmission instruction to a more communicative one, will allow pupils to recognise the value of mathematics in their every day lives. The engagement of children in mathematical problem solving has been key to all these initiatives.

2.8.1 Facilitating mathematical problem solving from an emergent constructivist perspective

Problem representation strategies are needed to process linguistic and numerical information, comprehend and integrate the information, form internal representations in memory, and develop solution plans (Noddings, 1985). These strategies facilitate translating and transforming problem information into problem structures or descriptions that are verbal, graphic, symbolic, and/or quantitative in nature. In turn, these representations assist in organising and integrating information as the student develops a solution plan. Specific representation strategies include paraphrasing or restating the problem in one's own words, visualising problems by drawing pictures, diagrams or charts, and hypothesising or setting up a plan to solve a problem (Montague and Applegate, 2000; Polya, 1945). Heuristics or problem solving procedures that incorporate these representation strategies are available for use in classrooms.

The most significant strategy was initially developed by John Dewey. His model of the problem-solving procedure may be described in four, five or six stages (Dewey, 1933). The following diagram describes the stages of the problem solving procedure and particularly emphasises why, for the problem solving process, it is important to engage in follow up or evaluation as it allows the student to examine whether the result satisfies initial conditions presented and also them to look ahead to form generalisations of both method and result.
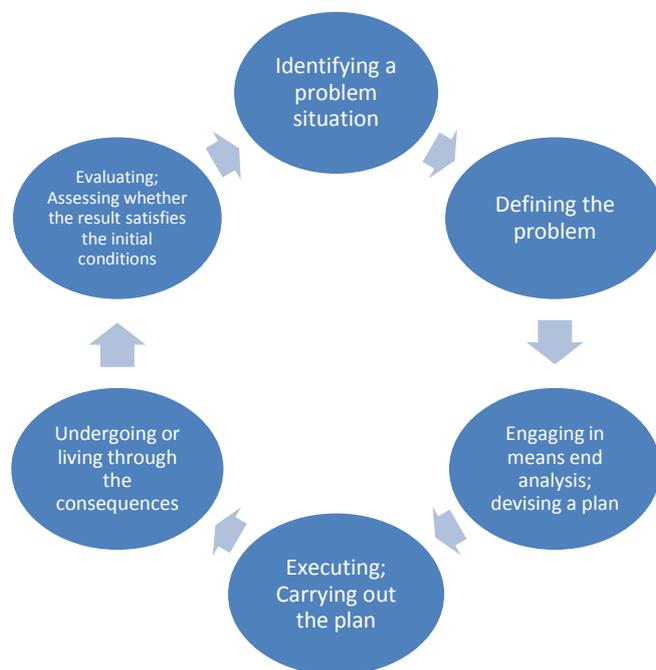
Figure 1: Dewey's model of the problem solving process

Noddings (1985) suggests that the first two stages have collapsed into one stage termed 'translation'. Since problems are presented to students in a predefined way, Noddings (1985) argues that the need to wrestle with the problematic situation has been removed from the process. Polya (1945) promoted a four stage model similar to that of John Dewey that was even more specific to mathematics and this model or heuristic is central to this thesis. Polya (1945) collapsed the first two stages and also eliminated the stage 'undergoing or living through the consequences'. It is this model that is used to foster mathematical

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thinking and develop students' ability to solve mathematical problems (Wilburne, 2006). The stages are typical of a socio constructivist learning environment in which ideas and strategies are shared with significant levels of experimentation and interaction. The following figure illustrates the four stage process.
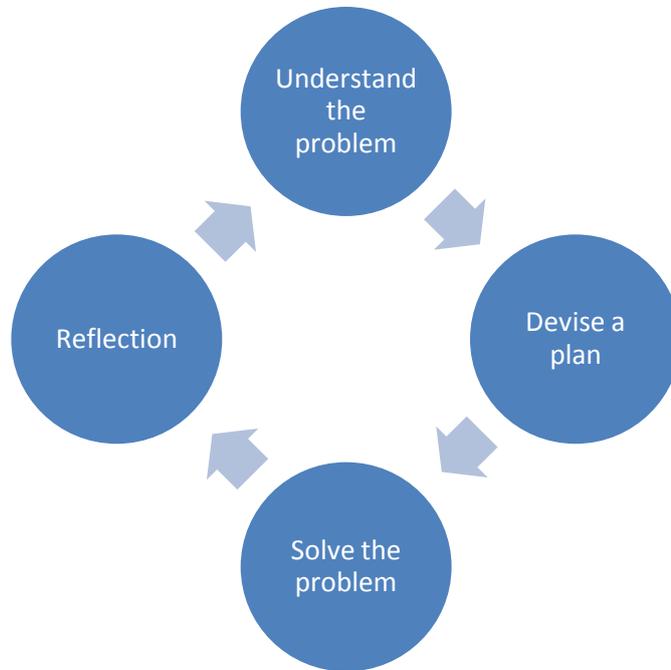
Figure 2: Polya's (1945) four stage problem solving procedure

In an elaboration on Polya's (1945) problem solving procedure, an example of a mathematical problem and student deliberations is threaded throughout. The problem is taken from a clinical study designed to investigate fourth class children's mathematical problem solving approaches (O'Shea, 2003).

A Frenchman revived the summer Olympics in 1896. The Olympic Games occur every four years. The table below shows information about the games. Use this information to create timelines for the summer and winter Olympic Games. How many times have the Olympic Games been held? How many more times have the summer Olympics been held than the winter Olympics?

Season	Start Date	Interval	Years Held	Years Cancelled
Summer	1896	4 Years	1896 – Present	1916, 1940, 1944
Winter	1924	4 Years	1924-1992 1994-Present	1916, 1940, 1944

(O'Shea, 2003:54)

2.8.1.1 Understand the problem

According to Rigelman (2007) and Wilburne (2006), at this stage students should be encouraged to come to terms with the problem by restating the mathematical problems in their own words and picking out the relevant information necessary to the solving of the problem. Discussion and debate is central to this stage of the problem-solving procedure, particularly about what is being asked by the problem and also in attempting to describe the information given by the problem. At this stage, students should be encouraged to represent the problem in another way, perhaps through the construction of a picture or a diagram or even by recalling whether they have previously solved a similar problem and how it was solved. Activities at stage one should be carried out by children in a group situation, as problem-solving itself is most efficient when done co-operatively with free opportunity for discussion. The above problem provided for children capitalised on children's natural sense of curiosity as, at the time of the study, the winter Olympic Games were underway in Salt Lake City (O'Shea, 2003). Therefore, discussion around the theme of the problem was easily ignited and children understood what was being asked of them by the problem.

2.8.1.2 Devise a plan

At this juncture, a connection between the data and the unknown is investigated and whether the operations to be made are known, giving the students a plan (Polya, 1945; Arslan and Altun, 2007: 51). If students are not sure of a connection between the data and the unknown, they should simplify the problem or solve part of the problem (Polya, 1945). Students should discuss the strategies that may be used to solve the problems and be encouraged to use any one or a combination of them that they deem suitable (Wilburne, 2006). The Primary Mathematics Curriculum (Government of Ireland, 1999a: 39) highlights the following strategies that are suitable for solving mathematical problems: drawing pictures, acting the problem out, using models, searching for pattern, making tables or charts, breaking the problem into smaller more manageable parts, writing equations or number sentences, using logical reasoning, guessing and checking, using mathematical equipment, working backwards from a solution, making lists and solving similar simpler problems. This list is quite exhaustive and contains the suggested strategies appropriate for use with elementary school students (O'Connell, 2000).

An element of flexibility should be encouraged at this stage since, as in everyday life, plans are merely rough guides that never uniquely determine future actions (Coll and Chapman, 2000). Roth and McGinn (1997) discuss this in relation to Geena, a cook, and provide an appropriate analogy to describe why an element of flexibility must surround any plan. Geena had an original recipe she had picked out for the purpose of baking cookies. Geena found that her original recipe would not have guaranteed the cookies she wanted to bake and therefore continued using the original recipe as a basis but also used her situated knowledge of baking and used other ingredients available in her setting. Considering again the above problem, at this stage, students decided to 'list all possible dates including irrelevant ones from 1896 to today' (O'Shea, 2003). Students at this stage of the problem solving procedure begin to develop a sense of ownership of the mathematical activity which enhances the building of personal meaningful mathematical understandings and students' confidence in their abilities (Francisco and Maher, 2005).

2.8.1.3 Solve the problem

As students solve the problem, they must be encouraged to consider each step in the process and be aware that they must be able to justify the specific step (Polya, 1945). During this stage, the solver is encouraged to think of ways of improving its accuracy (Polya, 1945). When answering the above problem, students utilised the facts of addition to calculate consecutive dates and listed them vertically then returned to the subsequent list of dates and erased years when the Olympic Games were cancelled (O'Shea, 2003). This is illustrated in the following diagram.
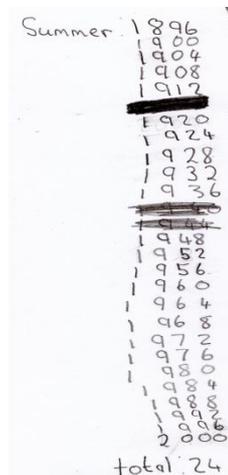
Figure 3: Stage 3 of Polya's (1945) problem solving process (O'Shea, 2003)

2.8.1.4 Reflection

Arslan and Altun (2007) explain that during reflection the solution is checked in terms of the original problem. Students are encouraged to justify the solution that they have arrived at.

> By looking back at the completed solution, by reconsidering and re-examining the result and the path that led to it, they could consolidate their knowledge and develop their ability to solve problems. A good teacher should understand and impress on his students the view that no problem whatever is completely exhausted. There remains always something to do; with sufficient study and

penetration, we could improve any solution, and, in any case, we can always improve our understanding of the solution (Polya, 1945: 15).

Alternative solutions and related problems, which can be solved by this strategy, should also be considered (Polya, 1945). Polya (1945) explains that just as we prefer perception through two different senses, we prefer conviction by two different proofs (Polya, 1945: 15). Students must also be encouraged to think of how they might apply the procedure used or the result obtained in another situation, because if students have no frame or reference for some aspect of a problem they may be unable to proceed (Hart, 1993). The final stage of Polya's (1945) four stage problem-solving procedure, the stage of reflection, is designed so that students and teachers may address such questions as: Is there a better way to solve this problem? Can the problem be generalised or extended? This stage will allow you to link this problem into your whole mathematical knowledge network with a view to finding a nicer solution of this current problem and storing up knowledge to tackle future problems (Hart, 1993). This is central to learning from an emergent perspective, recognising what the extensions are following engagement in the activity. With regard to the above problem, students discussed the merits of solving the Olympic Games problem in the manner it was solved. Students discussed the length of the solution and discussed whether or not it would have been more appropriate to subtract the initial starting date (in the case of summer this was 1896) from the date of the most recent games (in the case of the summer games it was 2000) and divide by 4 as this was the interval. Students decided that such a solution would not work in this case as the interval changed for the winter games from 1992 to 1994. According to Greer (1997) that when students take it upon themselves to question each others ideas and assumptions it helps them become flexible in future problem comprehension.

2.8.2 Engaging students in co-operative learning

Central to learning from a constructivist perspective is engaging pupils in co-operation and collaboration. Reform efforts aimed at improving mathematics education characterise the teacher's role as that of a facilitator supporting

students' learning as the engage with one another (NCTM, 2000). These efforts involve the teacher in guiding classroom mathematical practices and students' mathematical activity (Ball, 1993, Cobb, Wood and Yackel, 1993). This requires a sense of knowing on the part of the teacher as he/she attempts to capitalise on opportunities that emerge for mathematical learning from student interactions with one another (McClain and Cobb, 2001). Reform efforts also necessitate the monitoring of student practices and discussions as they engage with mathematics, and the provision of appropriate guidance as it is deemed necessary. To foster the growth of connections, interactive working, and discussion is more appropriate than solitary study (Davis and Petitt, 1994). However, teachers have not displayed enthusiasm for using co-operative learning methodology. The Primary Curriculum Review (NCCA, 2008a) reveals that whole class teaching is the organisational strategy most frequently used by teachers. This review also reported that teachers have yet to embrace fully co-operative learning as a primary strategy of instruction even though, as O'Shea (2002) found, co-operative learning situations are important since they encourage weaker pupils and simultaneously provide a medium for all pupils to adapt the problem-solving strategies that they possess.

Johnson and Johnson (1987) compared the achievements of people working alone versus co-operatively, and found that co-operative learning resulted in superior performance in more than half of the studies; in contrast, working alone resulted in improved performance in fewer than 10 per cent of the studies. Co-operative groups can foster achievement, motivation and social development. Palinscar, Brown and Campione (1993) explain that teachers should arrange learning exercises in which students are encouraged to assist each other. Davis and Petitt (1994) specify that these cooperative learning exercises should involve students explaining their understandings to their peers, trying a variety of ways to solve mathematical problems, and comparing their achievements with their peers. The less competent members of the team are likely to benefit from the instruction they receive from their more skilful peers, who, in turn, benefit by playing the role of the teacher (Palinscar, Crown and Campione, 1993). In mathematical problem solving this may be achieved by employing a

mathematical problem solving heuristic such as Polya's (1945) four stage heuristic.

Johnson and Johnson (1987) report different reasons for the effectiveness of co-operative learning. Students are more motivated when working on problems together; cooperative learning requires children to explain their ideas to one another and to solve conflicts. Engaging with a mathematical problem in a collaborative context helps young collaborators to examine their own ideas more closely and to become better at articulating them so that they can be understood. Also, children are more likely to use high quality cognitive strategies while working together – strategies that often lead to ideas and solutions that no one in the group would have been likely to generate alone (Johnson and Johnson, 1987). However, teachers should be aware that, children accustomed to classrooms in which they work alone can find it difficult to adjust to co-operative learning (Rogoff, 1990), although it has been found that they can get better with practice (Socha and Socha, 1994). Significantly though, as the structure of the school changes to support peer collaboration, with teachers' assuming roles of active participants in the children's learning experiences rather than simply directors of it, the benefits of co-operative learning are sure to increase (Rogoff, 1990).

2.9 Constructivism and mathematics teaching: Conclusion

Constructivist theory has had major influence on contemporary science and mathematics education. In the publication of the draft standards of the 1996 US National Science Education Standards, the contribution of the philosophy of science was essentially constructivist (Matthews, 2000). The NRC (1996) highlighted that implementing the standards would require major changes in much of science education. The standards are based on the premise that science is an active rather than a passive process. Learning science is something that students do, not something that is done to them. 'Hands-on' activities, while essential, are not enough. Students must have 'minds-on' experiences as well (Matthews, 2000).

As mathematics is also a science, to really engage with it, students need to be able to call on various mathematical skills as required. In developing the ability to do so, presenting or exploring isolated units of information is not desirable. Constructivist teaching allows for the development of such skills simultaneously, allowing the student to be an apprentice in the craft of mathematics. Fosnot (1989) believes that students need to think and learn for themselves. She abhors the fact that students of past curricula have been powerless in their own learning and largely dependant on the institution of the school. Fosnot (1989) may have been premature in realising that such powerlessness has been resigned to the past. In its fullest sense, the employment of constructivism in the classroom requires the personal and social construction of mathematical knowledge following the guidance of the teacher who follows culturally acceptable mathematical traditions. With the teacher as custodian of knowledge, the student constructs his/her own understandings in interaction with peers. As the current mathematics curriculum (Government of Ireland, 1999a; 1999b) purports to be centred on constructivist teaching and learning, and as recent research highlights the heavy focus by Irish primary teachers on closed-ended textbook problems (O'Shea, 2003) and little use of group mathematical activity (O'Shea, 2003; NCCA, 2008), there is a chasm between what is suggested by the curriculum and what is taking place in the classroom. This is due to the inherent difficulties in translating what is a theory of learning into consequences for teaching. However, by employing a heuristic or problem solving strategy as described above, teachers can begin to translate this theory of learning to a theory of teaching.

The final section of this review of literature focuses on mathematics education issues in the Irish situation. It pays particular attention to what Irish students have achieved and are achieving in relation to mathematical problem solving. It also takes a further look at the Irish primary mathematics curriculum as it is basis for a primary teachers work in the classroom.

2.10 Mathematics education: From an Irish perspective

The Primary School Mathematics Curriculum (Government of Ireland 1999a; 1999b) stems from its predecessor, Curaclam na Bunscoile (Government of Ireland, 1971. Curaclam na Bunscoile (Government of Ireland, 1971) was heavily inspired by Piagetian research and thinking. This curriculum emphasised particularly the individual nature of development of each individual child. It placed strong emphasis on first hand and concrete experience. In stating that 'he (the child) should be afforded opportunities to explore mathematics by the use of materials from his environment and by using structural material where necessary, so that he develops the concepts of mathematics in a meaningful way, through his own activity' (Government of Ireland, 1971:125) the curriculum reveals its solid Piagetian basis. It specifies the chief responsibility of the teacher as being 'to see that the pupils learn through their own discoveries rather than through information imparted to them' (Government of Ireland, 1971:125). Therefore, the Primary Mathematics Curriculum of 1971 was constructivist, in revealing the role of the teacher as being one of a guide or consultant it is easy to distinguish that is the philosophical basis for the Revised Primary Curriculum of 1999.

The current curriculum (Government of Ireland, 1999a; 1999b) is built upon the Piagetian foundations of Curaclam na Bunscoile (1971). It adds a particular social element and is quite clear on its constructivist basis. In its description of a child centred curriculum, the curriculum encourages the use of constructivist approaches.

> Constructivist approaches are central to this mathematics curriculum. To learn mathematics children must construct their own internal structures. As in reading and writing, children invent their own procedures (Government of Ireland, 1999a: 3).

The curriculum (Government of Ireland, 1999a; 1999b) acknowledges that children must experience formal mathematical instruction: 'We accept that children must go through the invented spelling stage before they begin to develop a concept of the structure of spelling. The same is true of mathematics'

(Government of Ireland, 1999a:3). However, the curriculum also states that, ultimately, the child should be encouraged to experiment with personal strategies, refine them through discussion and engage in a wide variety of tasks (Government of Ireland, 1999a). The curriculum advocates that the children be encouraged to operate in small groups or pairs to facilitate constructivist learning. Through involvement in these situations, children are expected to engage in the discussion of mathematical problems and their solutions, while supporting and helping other students.

The Mathematics Curriculum (Government of Ireland, 1999a) encourages children to adopt models of problem-solving behaviours. 'Children need to work out when to use a particular plan, what they want to achieve and the actual procedure needed to complete the task' (Government of Ireland, 1999a: 4). This coincides with literature that emphasises the significance of applying models of problem-solving behaviour during activity (Garofalo and Lester, 1985; Polya, 1945; Shavelson, Mc Donnell and Oakes, 1989). The curriculum acknowledges the importance of focussing on the process, as opposed to the product, as a medium of developing individual learning strategies. It also emphasises the use of open-ended problems, where considerable emphasis is placed on discussion and the acquisition of skills and not just the achievement of the correct answer. This is in line with current curricular thinking in the United States as outlined in Principles and Standards for School Mathematics (NCTM, 2000).

Von Glasersfeld (1989) explains that curricula could be designed with internal coherence and, would be more effective if they deliberately separated the task of achieving a certain level of performance in a skill from that of generating conceptual understanding within a given problem area (von Glasersfeld, 1989). Importantly, the mathematics curriculum (Government of Ireland, 1999a; Government of Ireland, 1999b) does not seem to reflect cultural assumptions that mathematics is a fixed body of knowledge that needs to be learned. It provides primary teachers with opportunities the opportunity to organise learning from a constructivist perspective and in fact states specifically that mathematical problem solving should be explored from an emergent constructivist perspective

as revealed above. However, how a mathematics curriculum is intended to be implemented and how it is implemented may differ.

As has been revealed by the NCCA (2008) in their review of the implementation of the revised primary curriculum, Irish teachers struggle with fostering learning in collaborative or co-operative group situations. Therefore, the next section examines national and international assessments of Irish primary and post primary students in an effort to reveal Irish students' competencies in relation to mathematical problem solving in particular. It is an examination of these competencies in relation to mathematical problem solving that will help uncover what is happening in Irish classrooms.

2.10.1 Trends in International Mathematics and Science Study (TIMSS 1995)

The Trends in International Mathematics and Science Study (1995) was an international assessment of the mathematics and science knowledge of students at five grade levels in over forty countries. Based in Boston College and conducted under the auspices of the International Association for the Evaluation of Educational Achievement in Ireland, it found that Ireland ranked nineteenth out of 25 countries in relation to mathematical performance (Mullis, Martin, Beaton, Gonzalez, Kelly and Smith, 1997). The study focussed on two age groups, 9-year-old primary students and 13-year-old secondary students. The mean scores of students on the mathematical ability tests administered showed that Irish pupils at fourth class level ranked well above the OECD average but at second level, Irish pupils ranked within the OECD average (Mullis, Martin, Beaton, Gonzalez, Kelly and Smith, 1997). TIMSS (1995) figures highlight a decrease in the level of mathematical ability between the senior grades at primary level and the initial grades at post-primary level. During the senior grades at primary level, and the initial years at post-primary level, instruction begins to focus more on problem-solving and the student's ability to combine his/her knowledge of the discipline with the information supplied in the problem. Students are not equipped to utilise their skills of analysis, prediction, estimation, and evaluation which are central to constructivist explorations.

The focus on scores in programs of national assessment and the focus and comparisons made following the publication of standards and achievements, such as in TIMSS (1995), often lead to debate about the degree to which the student can explain the thinking behind an answer. Data revealed by TIMSS (1995) is particularly interesting here. Cognisant of the fact that this was prior to the introduction of the Primary School Curriculum (Government of Ireland, 1999a; 1999b), TIMSS (1995) data highlights that fewer than 40 per cent of fourth class students in Ireland had teachers who felt it was important to think creatively, with 52 per cent of students being required to practice computational skills during most lessons (Mullis, Martin, Beaton, Gonzalez, Kelly and Smith, 1997). This is a particularly important statistic in the case of conducting mathematical problem solving classes from a constructivist learning perspective as creativity and experimentation is critical to the process. Such statistics imply that mathematical problem-solving activities are used solely to provide a context for the repeated practice of individual skills.

2.10.2 Programme for International Student Assessment (PISA)

The Programme for International Student Assessment (PISA), coordinated by the Organisation for Economic Cooperation and Development (OECD), tests and compares school children's performance across 57 countries. PISA assesses 15-year-old students' performance on 'real-life' tasks that are considered relevant for effective participation in adult society and for life-long learning. This is reflected in their definition of mathematical literacy. The OECD (2003:156) defines mathematical literacy as 'an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen'. This reflects the importance of problem-solving skills and constructivist philosophy: the individual must be able to utilise all mathematical skills and concepts acquired in the mathematics classroom in real-life situations and understand the use of such mathematical skills and concepts.

Student achievement is categorised at the various levels illustrated in Table 1. Tasks at Level 1 are associated with a minimum level of mathematics achievement, such as the ability to recall basic multiplication and division facts, the ability to read and interpret simple graphs, charts, scales and diagrams, and the ability to solve simple problems involving multiplication and division. Levels 2 and 3 are associated with students who have achieved a moderate level of mathematics achievement. At this level, students are engaged in basic reasoning, using problem-solving strategies, and linking symbolic structures to real world situations. At the upper end of the scale, at levels 5 and 6, students have an advanced level of mathematics achievement. Students at this level can develop their own novel approaches to problem solving, can select, compare and evaluate solution methods for solving problems, can communicate their mathematical ideas, and can discuss and compare their own mathematics with the mathematics of others.

The following table, taken from Eivers, Sheil and Cunningham (2007:27), illustrates the proficiency levels on the combined mathematics scale in PISA 2006, the percentages of students achieving each level, and compares the Irish scores to the OECD average.

Table 2: Proficiency levels on the combined mathematics scale in PISA 2006, and percentages of 15 year old students achieving each level (Ireland and OECD average)

Level Cut-Point	At this level, a majority of students can	IRL %	SE	OECD %	SE
Level 6 Above 669.3	Evaluate, generalise and use information from mathematical modelling of complex problem situations	1.6	0.25	3.3	0.09
Level 5 607.0 – 669.3	Develop and work with mathematical models of complex situations	8.6	0.67	10.0	0.12
Level 4 544.7 – 607.0	Work with mathematical models of complex concrete situations	20.6	0.94	19.1	0.16
Level 3 482.4 – 544.7	Work in familiar contexts usually requiring multiple steps for solution	28.6	0.90	24.3	0.16
Level 2 420.1 – 482.4	Work in simple contexts that require no more than direct inference.	24.1	1.00	21.9	0.17
Level 1 357.8 – 420.1	Work on clearly defined tasks in familiar contexts where all relevant information is present and no inference is required	12.3	0.93	13.6	0.15
Below Level 1 <357.8	Not respond correctly to more than 50% of Level 1 questions. Mathematical literacy is not assessed by PISA.	4.1	0.50	7.7	0.14

The above reflects the findings of such assessments as National Assessment of Mathematics Achievement (Shiel and Kelly, 2001; Surgenor et al., 2006) and TIMSS (1995) (Mullis et al., 1997). Irish students are proficient at tasks associated with levels 1-3, yet do not compare as well when dealing with mathematical reasoning and developing approaches to analysing, evaluating and

working with complex mathematical problems (levels 4-6). According to the OECD (2006), at levels 5 and 6 Ireland fares slightly less well than the OECD average and considerably poorer than countries such as Korea and Hong Kong where over 27 per cent of students reached level 5 or higher.

2.10.3 National assessments of mathematical progress

Nationally, a series of assessments have been conducted in relation to primary mathematics dating back to 1977. Currently, they are conducted by the Educational Research Centre, Dublin and they focus on children at various levels both at primary level and second level. These assessments have consistently shown that Irish primary students perform well in areas such as understanding and recalling basic terminology, facts, and algorithms, but not as well in problem-solving and engaging in mathematical reasoning (Shiel and Kelly, 2001; Surgenor et al., 2006). These conclusions are similar to the findings of TIMSS (Mullis, Martin, Beaton, Gonzalez, Kelly and Smith, 1997).

Earlier assessments involving students in second, fourth and fifth classes focussed on number, and these indicated that pupils were strongest in dealing with operations with whole numbers, and weakest in the area of problem-solving (Shiel and Kelly, 2001). This is consistent with current research into the achievements of primary pupils in Irish classrooms (Shiel and Kelly, 2001; Surgenor et al., 2006). Evidently, teachers place less emphasis on the teaching of problem solving than on the teaching of number (Shiel and Kelly, 2001; Surgenor et al., 2006).

Research into the teaching of mathematical problem-solving in Ireland is timely and due in no small part to the conclusions drawn by Shiel and Kelly (2001) and Surgenor et al. (2006). Both national assessments make for worrying reading particularly for those who emphasise the need for a focus on higher level mathematical processes such as problem-solving. The reports found students performed least well when engaging in mathematical reasoning, analysing, solving problems, and evaluating solutions, and understanding and making connections between mathematical concepts and processes (Shiel and Kelly,

2001; Surgenor et al., 2006). Furthermore, Shiel and Kelly (2001) explain that few schools have policies in place that place particular emphasis on the development of strategies for teaching problem-solving.

The Primary Curriculum Review (NCCA, 2008a) reveals that teachers reported challenges with the development of higher order thinking skills. Not surprisingly, Shiel and Kelly (2001) stress the need for a more intensive focus on higher level mathematical skills, such as problem-solving in schools. Worryingly, according to Surgenor et al. (2006), 90 per cent of inspectors concluded that they were only either 'dissatisfied'/'somewhat satisfied' with pupils' performance in engaging in mathematical reasoning, and 51 per cent reported dissatisfaction with the achievement of pupils in analysing and solving problems and evaluating solutions (Shiel and Kelly, 2001). Both these reports posit limited student proficiency in performing higher level mathematical operations. It may be that teachers are inhibited by the teaching methodologies they have acquired in developing pupils' higher order mathematical processes.

From a constructivist perspective, the employment of group collaborations in classrooms is essential, but 50% of inspectors involved in this survey expressed dissatisfaction with arrangements for grouping for mathematics in single grade fourth classes (Shiel and Kelly, 2001). The Primary Curriculum Review (2008) also found that teachers reported challenges with using collaborative learning strategies. Furthermore, Shiel and Kelly (2001) revealed that teachers were reluctant to see the calculator introduced to the primary school and that computer software was not be relied upon for teaching mathematics (Shiel and Kelly, 2001). The advent of technology allows students to spend more time on the process of problem-solving, since the calculations can be turned over to the machine (Williams and Shuard, 1982).

Overall, TIMSS (1995), PISA (2006), and both the 1999 and 2004 National Assessments of Mathematics Achievement (Shiel and Kelly, 2001; Surgenor et al., 2006) illustrate that Irish students perform well when presented with basic mathematics requiring them to use operations and recall basic facts and algorithms, but are challenged when it comes to using higher level mathematical

processes, including developing and working with novel mathematical problems and using their own methods and strategies in evaluating and solving mathematical problems. A chasm therefore exists between exceptional performance with basic mathematical facts, algorithms and operations and somewhat limited performance in higher level problem solving processes. This highlights the need to examine how students' higher level mathematical process may be fostered by teachers in classrooms that, at present, focus very much on basic, traditional mathematics.

2.11 Conclusion

The literature discussed in this chapter focussed on the themes of constructivism, mathematical problem solving and mathematics within the Irish context for specific reasons. Since the introduction of the primary curriculum in 1999, teachers have been attempting to teach mathematical problem solving from a constructivist perspective with little success. This lack of success has been due to insufficient levels of understanding, on the part of the teacher, of the implications of the constructivist theory of learning for mathematics teaching and particular challenges associated with reculturing classrooms towards a constructivist perspective. To address this, an attempt has been made to trace the origins of constructivist theory and elaborate particularly on the emergent perspective on constructivism as this perspective is inherently linked with mathematical problem solving. By adopting the emergent perspective on constructivism in the classroom, teachers must arrange learning situations where students debate, analyse, critique and defend mathematical problem solutions. Such activities are at the core of the work of mathematicians. By employing a mathematical problem solving heuristic in the mathematics classroom, teachers can facilitate learning from a constructivist perspective as all activities associated with problem solving heuristics are closely linked with the emergent constructivist perspective on learning. Mathematical problem solving classrooms are complex highly energised environments and by harnessing this energy and using it to motivate students to solve more complex and difficult problems, teachers can facilitate students in becoming real mathematicians, in

becoming problem solvers. The next chapter examines the research methodology chosen to engage in this research. It describes the detail of the research question and reveals how the research was investigated.

Chapter 3
Research Methodology

3.1 Introduction

This chapter examines the research methodology employed to engage in this research. It outlines the timeframe of the research in great detail and the specific tools utilised in data collection and approach to data analysis. This chapter explains the rationale behind the research question and therefore reveals why the specific methodology was necessary. The researcher chose to utilise the case study as the primary instrument of research. Data collection methods subsequently included semi-structured interview, observation, and audio taping.

The primary objective was to examine the engagement of students in learning mathematical problem-solving from a constructivist perspective. Utilising case study to achieve this, the researcher followed six primary teachers as they implemented constructivist learning theory in their mathematical problem solving classes. The participants are described are discussed in detail in chapter four. Research participants designed and developed templates for a mathematical problem-solving lesson, utilising Polya's (1945) four stage procedure as an initial starting point, from a constructivist perspective before going on to apply this template to their own situations and teach a series of mathematical problem-solving lessons. Their progress and achievements were monitored by the researcher throughout a full school term culminating in a series of semi-structured interviews. Student responses to these mathematical problem-solving lessons were also collected and students engaged in group interviews with the researcher.

3.2 Research question

The research question is to what extent will an understanding of constructivism and its implications for the classroom impact on teaching practices within the senior mathematical problem solving classroom? From the research question then, a number of hypotheses emerge. They are

- What is the current understanding of senior primary school teachers of constructivism?

- What are the implications of constructivist teaching practices for the Irish primary classroom?

- What is the impact of engaging senior primary teachers in professional development and constructivism?

3.3 Research rationale

The research question was influenced by a number of current factors that determined the need to focus on mathematics problem-solving in the primary classroom.

- The Primary Curriculum (Government of Ireland, 1999a; 1999b) espouses constructivist principles: 'Constructivist approaches are central to the mathematics curriculum. To learn mathematics, children must construct their own internal structures' (Government of Ireland, 1999b: 3). Yet, the NCCA Primary Curriculum Review (NCCA, 2008a) found that teachers have difficulty in engaging children in co-operative group situations and have asked for assistance in the implementation and, particularly, in the use of methodologies other than direct instruction.

- Both the National Assessment of Mathematics Achievement (Shiel and Kelly, 2001) and Counting on Success: Mathematics Achievement in Primary Schools (Surgenor et al., 2006) established that although Irish children are strong on understanding and recalling terminology, facts and definitions, and implementing mathematical procedures and strategies, they are weak in engaging in mathematical reasoning, analysing and

solving problems, and analysing solutions (Shiel and Kelly, 2001; Surgenor et al, 2006).

- The TIMSS (Third International Mathematics and Science Study) 1995 (Mullis et al, 1997) involved an assessment of mathematics achievement among Irish school children. It highlighted that, at fourth class level, Irish students achieved test scores that ranked well above the Organisation for Economic Co-Operation and Development (OECD) average. However, at post-primary year two Irish students ranked in line with the OECD average (Mullis, et al., 1997).

- PISA (2006) revealed that Irish students do not perform as well as their international counterparts in relation to mathematical reasoning and in the development of approaches to analysing, evaluating, and working with complex mathematical problems (Eivers et al., 2007).

- Cobb, Wood and Yackel (1991) explain that an immediate implication of constructivism is that mathematics should be taught through problem-solving. Thompson (1985) argues that, from a constructivist perspective, any curriculum aimed at promoting mathematical thinking must, by its very nature, be problem based.   Mathematical problem-solving opportunities were chosen because they allow students to verbalise their mathematical thinking, explain and/or justify their solutions, resolve conflicting points of view, and develop a framework that accommodates alternative solution methods.

- Surgenor, Shiel, Close and Millar (2006), following a national assessment of the achievements of 4th class pupils which revealed Irish students perform poorly in relation to mathematical reasoning and problem solving, reveal that there may be value in piloting an approach to the teaching and learning of mathematics that has a strong emphasis on problem-solving. A recent review of international trends in mathematics education, carried out by Conway and Sloane (2005,) examined the

principles underlying Realistic Mathematics Education. This approach to teaching mathematics was considered to represent a move away from solving traditional textbook problems towards solving problems set in real life contexts, that allow pupils to deduce general mathematics principles and develop specific mathematics skills, in the course of discussing, exploring and solving problems. Realistic Mathematics Education involves putting mathematics into recognisable, real life contexts to allow the pupils to engage with the mathematics and generate solutions in a variety of forms, encouraging discussion in a more informal atmosphere while moving towards a more formal solution.

3.4 Case study

'Case studies are the preferred strategy when how or why questions are being posed and the focus is on a contemporary phenomenon within some real life context' (Yin, 1994: 27)

Therefore, case study was specifically chosen as a research method to answer the research question as understanding the implications of engaging children with mathematical problem solving from a constructivist perspective for teaching requires the researcher to live through the experience with the participating teachers. Case study was the appropriate methodology to use as, qualitative case study, according to Stake (1995), is characterised by the researcher spending time in the situation under study, in contact personally with activities and operations of the case, and reflecting and revising meanings of what is happening. For the purposes of this research, constructivist practices were investigated with six particular individuals and the goal of the research was to understand those particular cases. The researcher was particularly interested in both their uniqueness and commonality (Stake, 1995). The researcher entered 'the scene with a sincere interest in learning how they (teachers) function in their ordinary pursuits and milieus and with a willingness to put aside presumptions while we learn' (Stake, 1995:1).

Adelman, Kemmis and Jenkins (1980) argue that case studies exist in their own right as a significant and legitimate research method. Each participant in the study is situated in a unique context and employing case study methodology allows penetration into these situations in ways that are not always susceptible to numerical analysis. The case study reports and investigates the complex dynamic and unfolding interactions of events, human relationships and other factors in a unique instance. Geertz (1973) explains that case studies strive to portray what it is like to be in a particular situation; to catch the close-up reality and dense description of participants' lived experiences of, thoughts about, and feelings for a situation. The case study 'seeks to understand and interpret the world in terms of its actors and consequently may be described as interpretive and subjective' (Cohen, Manion and Morrison, 2000:181). Cohen et al. (2000: 181) explain that a case study 'provides a unique example of real people in real situations, enabling readers to understand ideas more clearly than simply by presenting them with abstract theories or principles'. While recent trends in mathematics education in Ireland including TIMSS (Mullis et al., 1997), PISA (Eivers et al, 2006) and NAMA (Shiel and Kelly 2001; Surgenor et al., 2006) have provided us with rich quantitative descriptions of the achievements of Irish primary mathematics students, a classroom perspective is required to understand the particular difficulties that have been highlighted by these reports with teachers, particularly in their engagement with helping children to understand mathematics and engaging students in mathematical reasoning and problem-solving.

It is incorrect to define case studies as unsystematic or merely illustrative; case study data are gathered systematically and rigorously. The researcher noted that Nisbett and Watt (1984) counselled case study researchers to avoid journalism, selective reporting, anecdotal style, pomposity, and blandness. Nisbett and Watt (1984) reveal that case study researchers often pick out the more striking features of a case and, therefore distort the full account in an effort to emphasise the more sensational aspects. Similarly, case study researchers often only select evidence that will support a particular conclusion, which therefore misrepresents the whole case (Nisbett and Watt, 1984). From the presentation of data in chapter four, it is clear that a rich vivid description of the individual participants has been

provided from both participant and student perspectives and that the researcher was not selective in any of the description. The description places the experiences of the participating teachers and students in a wider context that gives the researcher the opportunity to place the experiences of the participant within the broader context of the classroom, providing a complete snapshot for the reader.

Anderson (1990) argues that education is a process and that there is need, therefore, for research methods which themselves are process oriented, flexible and adaptable to changes in circumstances and an evolving context. The case study is such a method. This research looks at the implementation of constructivist principles in primary mathematics classrooms following detailed examination of constructivist methodology and the development of problem-solving lessons that espouse those methodologies. It focuses on the deliberations of teachers and their interaction with students throughout the period of research. Teachers involved in the research project are considered part of the research team as the complementary strengths of the various team members can provide the necessary basis for a successful case study (Anderson, 1990).

3.5 Professional development

'Teachers learn in their work settings which support that learning and consequently become a stronger richer source of learning for all' (Loucks-Horsley, Hewson, Love and Stiles, 1998: 195)

It is clear that enhanced teacher quality is strongly correlated with improved children's attainment (Day, 1999) and that teachers' professional learning needs to be supported throughout the teaching continuum (Day, 1999; Hargreaves, 1994). Professional development programs are sources of ideas for teachers. These ideas are there to be experimented with and therefore increase the potential for teacher development. In the Irish primary school context, teachers have been supported in their development since the introduction of the Primary School Curriculum in 1999 by sustained periods of in service education. This in-service education was specifically designed to support teachers in the transition

between Curaclam na Bunscoile (Government of Ireland, 1971) and the Primary School Curriculum (Government of Ireland, 1999a; 1999b). Darling Hammond (2000) and Elmore (1996) reveal that teachers' professional learning can serve various purposes, including supporting transitions in the teaching continuum, the implementation of new curricula, school development, and the professional development of teachers. The purpose of the professional development designed for this research purpose was two-fold: to support the implementation of the Primary Mathematics Curriculum (Government of Ireland, 1999a; 1999b) and to support the professional development of the individual teacher engaged in the research.

Loucks-Horsely, Hewson, Love and Stiles (1998: 192) specified that 'the inadequacies of curriculum materials for a diverse population are a problem, particularly with the movement to build new learning on the learner's experience and context'. Similarly, the Primary Curriculum Review (NCCA, 2008a) revealed that, even after engaging in professional development concerning the Primary Mathematics Curriculum (Government of Ireland, 1999a; 1999b), teachers still have difficulty in engaging children in the development of their higher order thinking skills. Therefore, before engaging teachers in professional development, it was clear that a distinct void existed in teachers' repertoire of skills in relation to employing constructivist teaching practices, which affected the detail and structure of the professional development initiative. Given the varied experiences and qualifications of Irish primary teachers, the researcher, therefore, chose to examine constructivist practices from a philosophical perspective and direct the course from here towards examining simple classroom constructivist practices. The complete course is available in Appendix A.

The first stage of designing a professional development initiative is to understand the professional culture and its importance (Day, 1999). A professional development culture is essential to changing norms of pedagogy and practice and this occurs when teachers examine assumptions, focus their collective experience on solutions, and support efforts on the part of everyone to grow professionally (McLaughlin, 1993). A successfully established community will ensure that energy and enthusiasm among participants remains strong throughout the period

of research. Little (1982) and Rosenholtz (1991) found differences between schools where teachers communicated their experiences to one another, experimented with new strategies, talked about innovation, and shared success and failures. From the outset, all of the teachers engaged in the research were encouraged to share their experiences and expertise as the project progressed. A culture was established by bringing all participants together and engaging with their fears and questions about mathematics education from a constructivist perspective, and establishing positive relationships between teachers and the researcher. It is important to build a professional culture. With a supportive culture, teacher's newly gained knowledge and skills may have a chance of having a lasting impact on their teaching practices (Loucks-Horsely et al., 1998). McLaughlin (1993: 98) has underlined the influence of the professional learning community

> Classroom practices and conceptions of teaching…emerge through a dynamic process of social definition and strategic interaction among teachers, students, and subject matter in the context of a school or a department community. The character of the professional community that exists in a school or department – collegial or isolating, risk taking or rigidly invested in best practices, problem solving or problem hiding – plays a major role in how teachers see their work and their students and in why some teachers opt out, figuratively or literally, while many teachers persist and thrive even in exceedingly challenging teaching contexts (Mc Laughlin, 1993:98).

The researcher chose to build a professional development community with the teachers involved in the research and to establish a network to nurture and develop the relationships between the individuals involved (Loucks-Horsley, 1998). Little (1993) explains that professional development communities thrive when collaboration, experimentation and challenging discourse are welcome. Teachers engaged in group work and collaborative reflection on methodologies and practices they employed in the primary school mathematics classroom in their teaching of problem-solving during the initial stage of the professional development. This continued through all sessions in the design, selection and examination of mathematical problems that would be suitable to the particular

classrooms, in the design of the framework of a mathematics lesson from a constructivist perspective, and in the analysis of successful co-operative learning environments. Sincere appreciation of the co-operation of participating teachers is recorded here, particularly because collaboration is fostered by finding sufficient time to do so. Teachers willingly gave of their time and expertise to contribute towards this project. As teachers attempted to implement constructivist methodology in their mathematical problem-solving classrooms, the researcher was always available for support and assistance.

Snyder, Lippincott and Bower (1997), in their analysis of the use of portfolio in professional development, suggest that the most effective method employed in the professional development of beginning teachers is a practice-oriented model where participants devise plans, implement them, and reflect upon what happens as a result. Essential are the utilisation of multiple sources of evidence such as observations, third party observations, student work, lesson plans, and other evidence that can be gathered about a particular situation (Haugh, 2001). While engaging with the researcher prior to their implementation of constructivist practices in their classrooms, a significant feature of teachers' preparation included an examination of mathematical problem-solving lessons conducted previously from a constructivist perspective by the researcher. As teachers engaged in their own explorations they were encouraged to reflect critically on all aspects of the experience, and communicate these reflections when in conversation with one another and with the researcher.

> 'Teaching is a process of making sense of practice through the construction and reconstruction of experience. It is a moral act, a process of staying open to questions that arise in practice and engaging in conversation and response to these questions' (Haugh, 2001: 329).

Professional development enables teachers to keep pace with the changing demands of education and society. The teaching profession has been restricted in the methodologies it uses by the nature and range of pre-service and in-service training. This is reflected in the conclusions of the research. Historically, teachers have been trained to perform their work in a technical manner.

'Teaching consists of complex sets of differentiated interpersonal interactions with students who may not always be motivated to learn in classroom settings' (Day, 1999: 20). Therefore it is important that attention is given to the needs of the teachers as well as the needs of the children they teach. Curricula and methodologies are constantly evolving; consequently, the education system must encourage learning amongst its providers. A significant need has been identified, particularly in relation to mathematics teaching at primary level in the Irish state, and this professional development initiative in relation to constructivist practices has been an attempt to provide a response.

3.5.1 Professional development initiative: Mathematical problem-solving and constructivism

Before describing the stages of the research beginning with professional development, the following graphic gives an idea of how the research was organised and conducted (Figure 4).
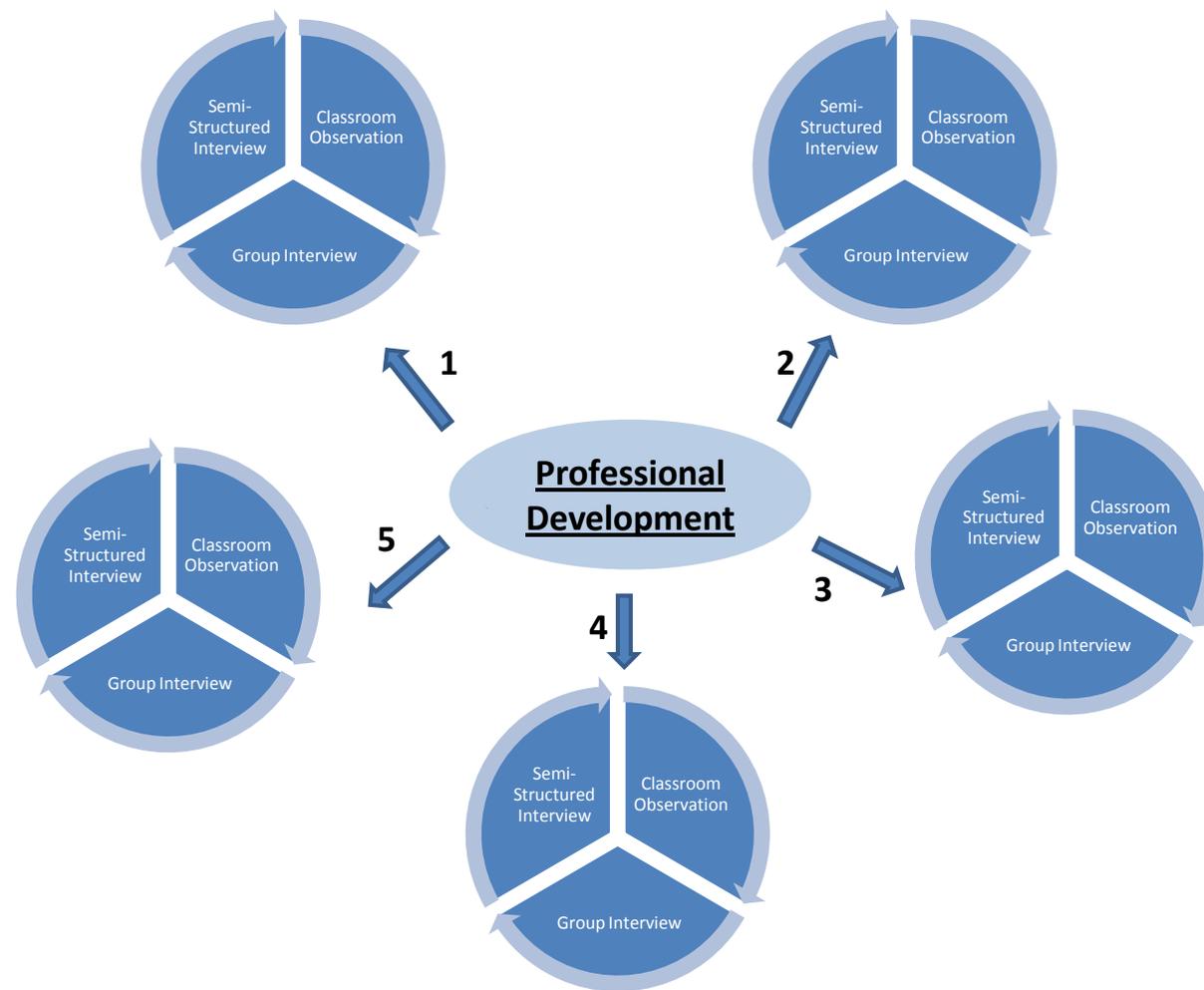
Figure 4: Overview of research

Initial sessions with participants focussed on mathematical problem-solving and constructivism. The following table presents an outline of this course, and all slides pertaining to all sessions are included in Appendix A.

Table 3: Outline of course for research participants

Session	Content	Brief Description
Session One	Behaviourism	Theories of Watson and Skinner, The Behaviourist Teacher
	Cognitivism	Modelling, Three-Stage Information Processing Model
	Cognitivism and Behaviourism: An Overview	Objective view of the nature of knowledge, Transfer of information utilising the most efficient means possible
	Introduction to Constructivism	Teaching for understanding, Critical Thinking, Authentic Learning, Child-Centred Curriculum, Active Learning
Session Two	Constructivism	Piaget, Vygotsky, Constructivism from a Radical, Social and Emergent Perspective, Learner Centred Education, Implications for the Primary Classroom
Session Three	Constructivism and Primary Mathematics	Primary School Curriculum (1999): Principles, Teaching for understanding or Training? Problem Solving Classroom: Personal Experiences, TIMSS (1995), Shiel and Kelly (2001), Surgenor, et al. (2006)
	Teaching from a Constructivist Perspective	Implications for the Teacher

Following this series of sessions the researcher visited every site and followed up on any queries or discussions any of the individuals had. In particular, detail was given on the types of problems that were chosen by the participants for exploration with their students and on the structure and design of a mathematical problem-solving lesson from a constructivist perspective.

Participants conducted a series of mathematical problem-solving lessons over a period from February to May 2008. Participants came from large, urban, mixed

primary schools and all participants taught one class of pupils only. The participants and their locations are detailed in Table 4.

Table 4: Participants, their locations and class levels

Participant	Class Level	Remark
Emily	5th Class	
Joe	6th Class	
Susan	6th Class	
Mike	6th Class	
Tomás	4th Class	
Jane	4th Class	Withdrew

Following their engagement in workshops, teachers agreed to teach mathematical problem-solving lessons that espoused constructivist philosophy and that were based on a core framework that all parties involved in the research agreed on, as follows:

- Starting points are to be real to the students. (Everyday scenarios used in the classroom can differ from those experienced by students outside of school.)
- Responses involving tables, drawings, diagrams, written explanations, constructing models were to be acceptable to all.
- Students are to be encouraged to explain their thinking.
- Students are to be encouraged to find different solutions to the problem.
- Students are to be encouraged to judge what counts as a different mathematical solution, an insightful mathematical solution, an efficient mathematical solution, and an acceptable mathematical solution .
- Students are to be encouraged to comment on the activity. Comments might include discussion on how the problem was solved, the nature of the discussion had by the group, the method of representing answers, etc.

3.5.2 Mathematical explorations

The following guidelines were established for the mathematical problem-solving lessons. Polya's (1945) four-stage problem-solving procedure was discussed and interpreted during introductory stages. These stages are

a)   understand the problem

b)   devise a plan

c)   carry out the plan

d)   look back.

The researcher in consultation with the participating teachers decided that utilising Polya's (1945) four stage problem solving structure would be significantly appropriate as a starting point for teachers who were only coming to terms with organising learning from a constructivist perspective. Polya's (1945) heuristic allows teachers to structure mathematical problem solving so that students play a large part in the development of strategies and solutions. Polya's (1945) heuristic also provides a forum where teachers can listen to and probe student understanding. These activities are central to a constructivist approach to teaching in the classroom.

3.5.3 Group work

It was agreed that the children would work in groups of mixed ability with an emphasis placed on active discussion surrounding the context of the problem, possible methods of solution, alternative methods of solution, and ways of presenting solutions to the problem. Individual group deliberations and problem solving activities were to be recorded for later analysis. The attempts of all groups in the classrooms at mathematical problem solving were not to be recorded on every occasion, rather one group from the whole class was chosen during each problem solving episode.

Small group problem-solving was used as a primary instructional strategy for the duration of the research. This was intended to give students the opportunities to participate in collaborative dialogue throughout the mathematics lesson, engaging with each other in the resolving of conflict in

relation to finding the solution to a task, which they were not accustomed to on a regular basis. The situations were initiated by the teacher and, cognisant of constructivist philosophy, the teachers were asked to guide the explorations of the students by for example, asking probing higher order questions. The teacher was actively involved with the students during group work by observing and questioning throughout the lessons. In a study of small group interactions in the classroom conducted by Cobb, Yackel and Wood (1991), teachers spent the entire time moving from one group to the next, observing and frequently intervening in their problem solving attempts. The interventions included encouraging co-operation and collaborative dialogue as well as discussing the solution attempts of the children (Cobb, Yackel and Wood, 1991). This was actively encouraged. Teachers were acutely aware of the differences between a facilitator and director and were encouraged to facilitate students learning in accordance with constructivist methodology, which was discussed prior to engaging in research with the students. The features of the mathematical lessons were non-routine mathematical problems. The activities were designed in partnership by the researcher and the teachers in order to stimulate students to engage in mathematical thinking and discussion. In accordance with constructivist methodology, the problems made reference to some experience of the students involved. Individual teachers assumed responsibility for identifying appropriate mathematical problems for the students with whom they worked. Teachers paid significant attention to the background understanding of their respective classes in attempting to identify mathematical problems that would suit both their experiences of data and facilitate them in their engagement with Polya's (1945) four-stage problem solving procedure. Following group activity the students explained and justified their work in whole class discussion.

3.5.4 Mathematical problems

Teaching activities espousing constructivist principles were conducted by participants with their students over a period of one school term. The primary purpose of these activities was to experience at first hand students' mathematical learning and reasoning. These activities used mathematical

problems that were deemed appropriate, by the researcher in association with the participant (class teacher), for use at the senior end of the Irish primary school.

When sourcing problems for children to solve in their groups, teachers were to be aware that it was not necessary that children would have seen a similar problem to the one chosen for them to investigate. This would ensure considerable discussion around the context of the problem, and require students to call upon knowledge that they had already in attempting to generate a plan or a solution method for their task at hand. It was emphasised that the method students might require to solve the problem was not to be prescribed and perhaps might not have been taught at all.

The mathematical problems chosen by the teachers varied in their content and spanned all strands of the Primary Mathematics Curriculum (Government of Ireland, 1999a; 1999b). The problems studied and explored with pupils are presented together with a transcription of the unfolding events of the class in chapter four. Participants spent time discussing their problems, examined each others mathematical problems, and reflected on the type of experiences they would provide for students. As the researcher was not in a position to identify the capabilities of every student involved in the research project, participating teachers were urged to use their professional judgement following reflection upon the implications for constructivist lessons and what might be particularly relevant in a problem to initiate meaningful discussion from a pupil's perspective.

3.5.5 Writing instructions

The importance of recording all of the activities that they were engaged in was made clear to students. They were encouraged to record their solutions in recipe format. The idea of a recipe was discussed with all pupils. Both teachers and researchers made it clear to students that others might need to follow in their footsteps and solve problems using their solution methods, and therefore would need clear step by step guidance. Students were also helped to understand that

for their own information, and for developing strategies for solving problems of a similar nature in future, it would be beneficial to have a good record of the activities they engaged in the past. Students were supplied with copybooks designed by the researcher for this purpose. It was agreed that students would be more comfortable in reporting back to the class after the problem-solving situation was completed if they had a written description of the activities that they had engaged in to hand.

3.5.6 Researcher Visits

As teachers began to conduct mathematical problem-solving lessons constructively, one teacher requested the researcher to visit the school and model a typical mathematical problem-solving lesson. Following this, and throughout the period of the investigation, the researcher visited all schools and classes to gather research by taking field notes and audiotapes of the problem-solving sessions. During the final visits to the research sites students engaged in group interviews and participating teachers engaged in semi-structured interviews and with the researcher.

3.5.7 Access and permission

According to Cohen, et al (2000), social research requires the consent and co-operation of subjects who are to assist in investigations and of significant others in the institutions or organisations providing the research opportunities. The consent of every board of management was acquired (Appendix B.1), the consent of every teacher was acquired (Appendix B.3), the consent of every parent of every student involved was acquired (Appendix B.4), and the consent of every student was acquired (Appendix B.4).

Pivotal to the whole relationship between researcher and researched is access and acceptance (Punch, 1986). Cohen et al. (2000: 51) explain that 'the principle of informed consent arises from the subject's right to freedom and self-determination'. Diener and Crandall (1978) explain that informed consent is when individuals choose whether or not to participate in an investigation after

being informed of the facts that would be likely to influence their decision. The definition involves the elements of competence, voluntarism, full information, and comprehension. Competence implies that responsible, mature individuals will make correct decisions if they are given the relevant information. Voluntarism ensures that participants freely choose to take part (or not) in the research and guarantees that exposure to risks is undertaken knowingly and voluntarily. Full information implies that the participant is fully informed. The term 'reasonably informed consent' applies here as the researchers themselves do not know everything about the investigation. Comprehension refers to the fact that participants fully understand the nature of the research project, even when procedures are complicated and entail risks (Cohen et al, 2000). Participating teachers and principals of the schools involved were fully briefed in a letter (Appendix B.2) and also at an initial gathering prior to the commencement of the research project.

In the case of the children involved in the research, it is important to keep in mind that they cannot be regarded as being on equal terms with the researcher. There is a two-stage process involved in seeking informed consent with regard to minors (Cohen et al., 2000). This process was adhered to. Firstly, the researcher consulted, and sought permission from, those adults involved (parents, teachers, etc.) and secondly, the young people themselves were approached. The relationship implies a respect for the rights of the individual whose privacy is not invaded and who is not harmed, deceived, betrayed, or exploited (Burgess, 1989).

3.6 Research design

The following table gives an overview of the stages involved in this research from the point when participants agreed to take part in the professional development initiative to the final stages of the research.

Table 5: Timeline of events

Stage	Event
(1) Autumn 2007	Professional Development Initiative – Teachers engage with the researcher on the topic of constructivism and mathematical problem-solving over the course of three sessions
(2) Spring Term 2008	Researcher visits the research sites to engage in individual discussions with research participants
(3) Spring Term/Summer Term 2008	Participant teachers engage students in mathematical problem-solving from a constructivist perspective. Researcher visits the sites taking field notes and audiotapes of mathematical problem-solving sessions, and gathers documentary evidence
(4) Summer Term 2008	Researcher visits research sites and engages students in group interviews and teacher participants in semi-structured interviews.

3.6.1 Collection of Data

The data from the case studies was meticulously gathered in both oral and written format. Patton (1990:10) explains that qualitative data consists of 'direct quotations from people about their experiences, opinions, feelings, and

knowledge'. The participating teacher's mathematical problem-solving lessons from a constructivist perspective were recorded on audiotape for analysis. These recordings were designed to capture simultaneously the teacher's exploration of the mathematical problems and the attempts of a group of his/her students' at solving the problems presented to them by the teacher.

Interviewing is the most common from of data collection in qualitative studies in education (Merriam, 1998). The following details the semi-structured interview, the primary data collection method used by the researcher during the period of data collection.

3.6.2 Semi-structured interview

Anderson (1990) explains that interviews are prime sources of case study data. The format of the semi-structured interview was chosen for the purposes of data gathering. The semi-structured interview was particularly suitable as it is flexible and can be adapted to the personality of the person being interviewed. The semi-structured interview allows the researcher direct interaction with participants and facilitates greater depth of data collection. The semi-structured interview encourages two-way communication; those being interviewed can ask questions of the interviewer. Semi-structured interviews are conducted with a fairly open framework which allow for focused, conversational, two-way, communication (Anderson, 1990). This was necessary because of the approach taken to the project by the researcher in the initial stages. Communication between all parties was always encouraged with participants being asked to raise issues for discussion on regular occasions. The semi-structured interview helps the researcher to get answers to questions, and also to get reasons for those answers.

According to Cohen et al. (2000) 'semi structured interviews enable respondents to project their own ways of defining the world. It (the interview) permits flexibility rather than fixity of sequence of discussions, and it also enables participants to raise and pursue issues and matters that might not have been included in a pre-devised schedule' (Cohen, et al. 2000:147). Semi-structured

interviews are designed to 'develop ideas and research hypotheses rather than to gather facts and statistics' (Oppenheim, 1992: 67). Semi-structured interviews were utilised in this research to ascertain 'how ordinary people think and feel about the topics of concern to the research (Oppenheim, 1992: 67). Cohen, et al. (2000) suggests that 'interviews enable participants to discuss their interpretations of the world in which they live and to express how they regard situations from their own point of view'.

Semi-structured interviews allow for the exploration of unanticipated ideas that may arise during the course of an interview. Therefore, the interviews were guided by a series of broad questions (Appendix H). Questions were not asked in the order they are presented in Appendix H. Questions were asked in a natural way that was appropriate to the context of the discussion. The interview schedule did not determine the structure of the interview, but served as prompts for topics to be covered, provided the research with an agenda to follow, assisted in monitoring the progress of the interview, and provided logical and plausible progression through the issues in focus. All interviews were recorded on audiotape and transcribed for analysis. Every individual interview was transcribed for analysis by the researcher using Express Scribe.

There are many advantages in using the semi-structured interview as a research tool. The semi-structured interview is compatible with several methods of data analysis (Willig, 2008) and, therefore, was a suitable research tool with regard to this particular study. It enables the researcher to research hypotheses and develop ideas rather than just gather facts and statistics, and also assists the researcher in understanding how ordinary people feel about matters of concern to the research (Oppenheim, 1992). The semi-structured interview also allows for adaptability and flexibility (Opie, 2004) by enabling the researcher to explore issues in more depth and detail, allowing the interviewer to probe responses and gain a greater insight into the interviewee's life or experience (Willig, 2008). The researcher can also observe the interviewee, allowing him/her to investigate the motives and feelings of the interviewee and investigate the way the response is made by observing tone of voice, facial expression, etc. (Cohen, Manion and Morrison, 2000). Each of the key areas of

questioning can also be covered, allowing the researcher to gain optimum responses (Opie, 2004).

It is also worth noting that there are also disadvantages in using the interview as a research method. Interviews often reflect the beliefs and viewpoints of the interviewer and can be very subjective, with a danger of bias emerging (Opie, 2004). In this instance the researcher endeavoured to remain as objective and impartial as possible during the interview process. Interviewing about sensitive issues can prove difficult, and the relationship between the interviewer and interviewee can impinge on the questions asked and conclusions drawn (Opie, 2004). Taking the time to conduct the interview and analysing the interview and can also be very time consuming.

3.6.3 Group interview

In an effort to help students open up about their mathematical experiences in their classrooms, and their experiences of being in mathematical classrooms conducted from a constructivist perspective, the researcher engaged students in the classroom in group interviews. Group interviews are less intimidating for children (Lewis, 1992). Particularly in the case of child participants, group interview is a way of getting children to open up. The group interview can generate a wider range of responses than individual interviews. As the researcher was also a guest in the schools of participating teachers, group interview was used because they are quicker than individual interviews and involve less disruption (Lewis, 1992). Similar to the semi-structured interview, a menu of questions was constructed but these were intended as a guide rather than a prescription. Transcriptions of these interviews are available in Appendices C.2, D.2, E.2, F.2, and G.2.

3.6.4 Interview schedules

Interview schedules consisted of open-ended questions to encourage the exposition views, allowing for the development of thought and probing of responses. Robson (2002:41) explains that it is 'a shopping list' of questions.

Robson (2002) stresses the importance of including thought-provoking questions in semi-structured interviews. At all times participants were put at ease and were asked for permission for the researcher to record the interview. Semi-structured interviews were conducted with participating teachers in the participants' schools.

3.6.5 Quality of research design

Four tests are commonly used to establish the quality of any empirical social research. They are: construct validity, internal validity, external validity, and reliability. Table 6, adopted from Yin (2003), lists these tests and characterises them according to the phases and actions carried out during the research timeframe of this particular study.

Table 6: Case study tactics and responses (adapted)

Test	Case Study Tactic	Phase of Research during which tactic occurs	Actions taken
Construct validity	Use multiple sources of evidence	Data collection	Use of interviews and documentary evidence
	Establish chain of evidence	Data collection	Interview data both taped and transcribed in real time; multiple evidence sources entered into customized participant portfolios
Internal validity	Pattern matching	Data analysis	Patterns identified across cases
	Explanation building	Data analysis	Some causal links identified
Reliability	Use case study protocol	Data collection	Same data collection procedure followed for each case; consistent set of initial questions used in each interview
	Develop case study database	Data collection	Interview transcripts, other notes and documents collected and kept in participants portfolios
External Validity	Use replication logic in multiple-case studies	Research design	Multiple cases investigated using replication logic

3.7 Data analysis

Yin (2009: 127) has explained that 'the analysis of case study is one of the least developed and most difficult aspect of doing case studies'. It is important that, at the outset of engaging in case study, the researcher is aware of how the data is to be analysed. Before engaging in the research, and with experience of being a teacher of primary mathematics and problem solving, it was evident to the researcher that a number of different sources of data would provide a complete picture of events in the classroom, including both data gathered from both teacher and student. Therefore, analysis of data was going to be achieved by searching for rich patterns and themes across all of the evidence and subsequently across all of the cases. Computer assisted software was not used in the analysis of data gathered during research; rather, a general analytic strategy as suggested by Miles and Huberman (1994) was initially put in place. By creating arrays and creating categories, themes emerged from the data and these themes were explored, analysed, and are reported accordingly.

Yin (2009) explains four general analytic strategies. They are: relying on theoretical assumptions, developing a case description, using both qualitative and quantitative data, and examining rival explanations. The most preferred strategy is to follow the theoretical assumptions that led to the case study. The research rationale that was outlined at the beginning of this chapter has revealed the necessity for examining a constructivist approach to mathematical problem-solving and, together with the existing literature on teaching from a constructivist perspective, enabled the researcher to rely on theoretical orientation when engaging in analysis. For example, significant data exists on teacher attitudes and perspectives on incorporating change into the classroom; therefore, the researcher analysed the data to describe reactions and issues that arose when incorporating constructivist teaching practices in to everyday mathematics teaching practices employed by these five particular teachers.

Following data collection and the organisation of material into specific cases, the researcher, cognisant of the literature that has been outlined in chapter two, searched the data for particular themes and patterns from the observations, semi-

structured interviews, transcriptions of the mathematical problem-solving lessons, and pupils' work. Themes emerged from across the individual cases and indeed across the five case studies: a focus on rote memorisation, mathematical problem-solving from a constructivist perspective as enrichment activity, and teaching students with different learning abilities from a constructivist perspective. Furthermore, having multiple cases makes analysis easier and the findings are more robust than having one single case (Yin, 2009). Cross case synthesis was performed even though each individual case was treated as a single study. The individual cases are reported in chapter four, analysed in chapter five, and the cross case synthesis is performed in chapter six.

3.8 Triangulation

When researchers engage in investigation or research in the social sciences they attempt to explain in detail the complexity of the behaviour under investigation. Single observations provide a limited view of any behaviour because human interactions and behaviour are particularly complex, and also 'exclusive reliance on one method may bias or distort the researcher's picture of the particular slice of reality she is investigating' (Cohen, Manion and Morrison, 2000). The researcher has used multiple data collection methods in the completion of this research, together with examining multiple cases. Multiple case study research is more robust than a single study (Yin, 2009).

Patton (2002) discusses four types of triangulation: data, investigator, theory, and methodological. Two forms were employed in this study, data and methodological triangulation. The researcher used various data collection throughout the course of the research, including semi-structured interview, observation, audio recordings of the mathematical problem-solving lessons and student's written work completed during mathematical problem-solving lessons. Information was collected from multiple sources aimed at corroborating the same fact or phenomenon: this is data triangulation. The following diagram (Figure 5) attempts to show how the researcher endeavoured to produce convincing and accurate case studies.

Figure 5: Case Study Triangulation
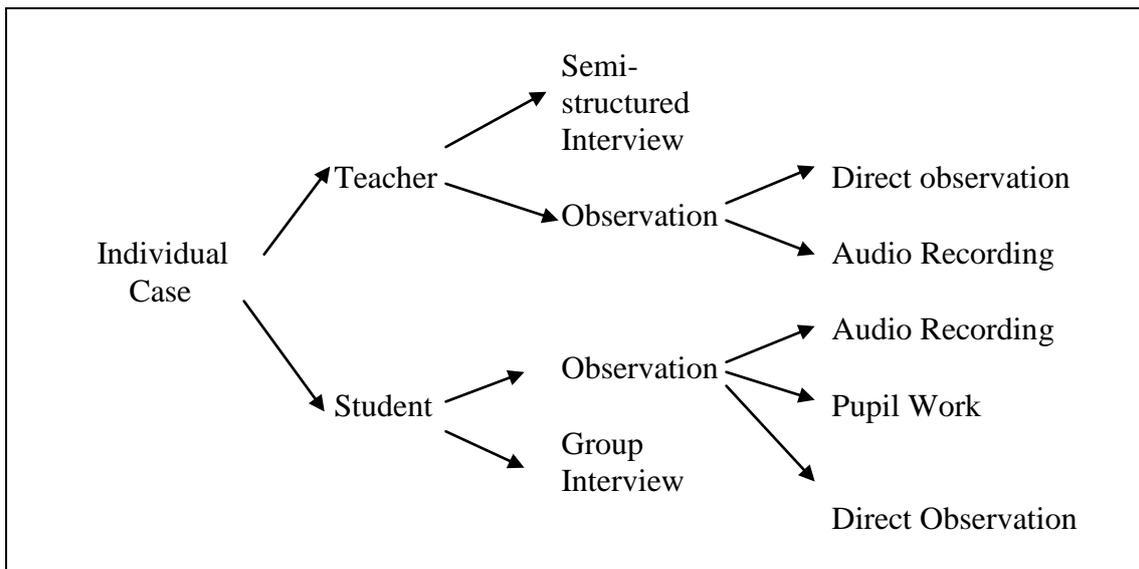
3.9 Conclusion

This chapter outlined the research methodology employed during the course of this research. It gave a comprehensive description of the design of the research and the professional development initiative engaged in by research participants. Chapter four presents the data gathered during research. Data is presented on a case by case basis and includes material gathered during semi-structured interviews with teachers, material gathered as teachers engaged with their students during the mathematical problem-solving lessons, documentary evidence gathered from students as they engaged with the mathematical problem-solving lessons, and material gathered from students as they engaged in group interviews.

Chapter 4

Presentation of Data

4.1 Introduction

This chapter presents data gathered throughout the course of this research. Data is organised according to the individual participating teacher and his/her students and, therefore, five cases are presented. As previously outlined in chapter three, data has been gathered through observation, semi-structured interview, and by document. Following the completion of their professional development and their exploration of mathematical problem-solving lessons from a constructivist perspective, teachers agreed to participate in semi-structured interviews. Participating teachers' students also agreed to participate in group interviews on completion of the mathematical problem-solving lessons. Complete transcripts of all interviews are available in the appendices and the relevant appendices are indicated as data is presented. The participating teachers' names have all been changed so that no individual can be identified. Similarly, students are identified by letters of the alphabet for this reason. The participating teachers involved in this project since it's inception are Susan, Emily, Joe, Tomás, and Mike and this is their story.

4.2 Participant one: Susan

Susan was a participating teacher in a Limerick city school. The following is a photograph of Susan's classroom

Photograph 1: Susan's classroom

4.2.1 Susan's profile

Susan is a sixth class primary teacher and has been in her current position for 2 years. She has taught at primary level for nine years having experience of teaching infant classes, middle classes, senior classes, teaching children in special educational needs situation, and acting as a home school liaison officer. She has both primary and masters degrees in education and has contributed to courses for undergraduate students of primary teacher education. She has a very open and vivacious personality and from the outset she was very enthusiastic about participating in the project. Prior to interview Susan asked if she could be controversial to which the researcher responded: 'You can say what ever you like' (Appendix C.1). This was evidenced in her contribution to the discussions that took place before the project was undertaken. Susan was very open to engaging in constructive discussion about the meaning of constructivism and its implications for her mathematics teaching.

4.2.2 Susan's teaching of mathematics

Susan has a real interest in mathematics, admitting she has enjoyed the subject since her own days as a primary school student. She continued to study higher level mathematics at second level, travelling to the local boys' secondary school for tuition because higher level mathematics was unavailable to her in her all

girls' secondary school. Susan stressed the need for higher level mathematics to be available to all pupils irrespective of gender: 'At second level now there was a big gender bias in our school from Leaving Cert so we had to go to the boys' school. So I think it must be provided for both genders' (Appendix C.1).

Susan firmly believes the enthusiasm and commitment of the teachers of mathematics at both primary and second level is crucial to developing a student's interest and passion for the subject. She felt that it was her teachers that instilled in her a passion for and commitment to the subject: 'It really depends on the teacher and their abilities. There are a lot of teachers that do the subject an injustice' (Appendix C.1). Susan went on to study mathematics at third level, which again reveals her interest in the subject. Susan criticised the manner in which she was prepared to teach mathematics at third level: 'I can remember sitting in a large group taking notes. We never had a practical maths session as in with equipment. We were shown things but never used them. That is a big problem I think' (Appendix C.1).

Susan revealed how her primary level teacher made mathematics interesting for her: 'It was very positive, there was a lot of solving problems, a lot of concrete materials, abacus and things like that' (Appendix C.1). Susan was a very able student at primary level and her teacher capitalised on her ability by using what Susan referred to as 'difficult textbooks' (Appendix C.1): 'At primary level we had *Busy at Maths* and *Figure It Out* and some follow up books. The brighter ones stuck with the *Figure It Out* and the others had the *Busy at Maths* books' (Appendix C.1). Susan revealed that a significant emphasis was placed on the learning of tables at primary level and this has stayed with her in her teaching.

> We learned our tables backwards inside out and it stood to me, I find that it's not the case today and that is a real problem. My experience with in-service is that it hasn't been advised which is a big mistake. There is a lot of skip counting now and children can't do their tables when it comes down to doing their sums in class. Or it is taking too long (Appendix C.1).

It is clear that Susan finds fault with recommendations made by the support service for the introduction of the primary curriculum. She revealed that the rote memorisation of tables was not encouraged at in-service days dealing with the introduction of the Primary Mathematics Curriculum (1999). When asked about the value of the rote memorisation of tables, Susan revealed that she believes primary students are at a distinct disadvantage if they do not know their mathematics tables extremely well: 'Children are not always going to have a calculator and you are not going to have a calculator inside in the shop. You need to be able to do mental maths' (Appendix C.1). Susan believes that there is a place for the calculator in primary mathematics but said, 'I think we should be careful that we don't over use it' (Appendix C.1). Susan explains that, in her opinion, there is too much skip counting.

Susan teaches a mixed gender sixth class. The class is considered academically weak at mathematics by both Susan and the school. All sixth class students are separated into streams for the teaching of mathematics. Resource teachers help with the teaching of mathematics specifically. Susan describes her class as follows:

> I think my class are a particularly different situation from others as the groups they are extremely weak and the vast majority are below the 20th percentile in the Drumcondra Primary Maths. They are drawn from other sixth classes as well. It is a stream, a good group an average group and a weak group (Appendix C.1).

Susan believes that the abilities of the students in her class were an inhibiting factor in the teaching of mathematical problem-solving from a constructivist perspective. Susan stressed that, because her students did not have a firm grasp of operations, approaching the teaching of mathematics from a constructivist perspective was particularly challenging.

Some of them have even difficulties adding hundreds, tens and units, and some of them had some idea about, for example, the addition of fractions so a very mixed bag indeed. Constructivism is great and I will do it next year where I know my class will

enjoy it more and get more benefit out of it but this year is particularly hard (Appendix C.1).

Susan knows the students that she will be teaching next year and, in discussion, explained that she will approach mathematical problem-solving from a constructivist perspective with them as she believes their mathematical ability will allow her to do so.

Susan explained that the students' interpersonal skills outside of the mathematics classroom were an inhibiting factor to their engagement with mathematical problem solving from a constructivist perspective: 'The children were very weak outside of maths I don't think they had the interpersonal or the group work skills needed to engage with it' (Appendix C.1). The focus of the majority of Susan's work with the students prior to their engagement with this project was basic computation involving the four operations, addition, subtraction, multiplication and division and basic computation involving simple fraction. Susan attributes her students' difficulties in approaching mathematical problem-solving from a constructivist perspective to the students' general lack of experience with any kind of problem-solving: 'It was a lack of problem-solving; they hadn't experienced enough of it, but where do you go if they can't add subtract or multiply' (Appendix C.1).

In discussion about the particular students Susan teaches she outlined further difficulties that impacted on the teaching of mathematical problem-solving from a constructivist perspective. The amount of students assigned to individual classes causes difficulties for Susan in the delivery of the Primary Mathematics Curriculum (1999) as it was designed.

> The problem here that I want to highlight is the inclusion of children with special needs in the class it is just impossible. I have had 33 in a class. There were at one time 6 working on a special curriculum which means they were maybe on first second or third class level. So there I am spending 10 to 15 minutes on a concept. We do a number of examples, I invite children up to the board and we talk it through I give them work to do. Then there is the problem I go to my other students who by then have no mathematics teaching in the 20

minutes that is already gone in my maths lesson. Invariably then someone out of my 27 will say I'm stuck. Now I can't tear myself up into 4 pieces and go around to them. It is just impossible to teach that many children (Appendix C.1).

Following reflection upon the academic strengths and weaknesses of her students, Susan chose to use a constructivist approach to mathematical problem solving with the class by choosing simple straight forward problems from which to work with but reveals that she still encountered difficulties: 'The problems themselves had to be very basic and even still then they caused problems' (Appendix C.1). Susan added that her students she had particular difficulties with long term memory: 'Their long term memory I feel was very poor indeed. Coming back from a break or the holidays was like they had never seen any of it before' (Appendix C.1).

As indicated, Susan places significant value on the rote memorisation of number facts. This belief in drill and practice and rote memorisation is also evident in Susan's every-day approach to the teaching of mathematics: 'Back to rote leaning it is very important. They need their facts' (Appendix C.1). Susan further elaborated explaining: 'We need to go back a little bit to the old style where it was drill. Tables for example need to be recited. They need drill. I don't let them use calculators regularly. They think it is just fun when they are allowed use them' (Appendix C.1). In discussion about Susan's daily approach to the teaching of mathematics, She explained that 'a typical daily lesson would start off with ten minutes mental maths involving everything, fractions, decimals and percentages. Then the main part of the lesson whatever they are doing, I do examples on the board and then the children do a lot of work' (Appendix C.1).

Susan explained that children spend quite a substantial amount of time working as individuals during mathematics lessons. Susan does provide opportunities for her students to work in pairs and also in group situations but stressed that, because of large class sizes, it does not happen on a regular basis. 'They work in pairs at times and then they do work in groups sometimes keeping it to a

maximum of 4 in a group because of logistics. They do a lot of individual work though' (Appendix C.1). Susan revealed that when solving problems with students she used group situations. When asked about what a good mathematics student can do, she explained that a good mathematics student can 'use trial and error, explain the thinking behind the sums, knows their tables and can do mental maths very quickly' (Appendix C.1).

When asked if she ever tried anything she might consider different or outside of the norm in her teaching of mathematics, Susan explained that she had engaged her students in mathematics trails and games but not on a regular basis. She also explained that the internet had become a great source of ideas and mathematical problems for her in planning for and teaching of mathematics.

Susan has a fundamental belief in drill and practice and rote memorisation stemming from her own experiences in primary school. In discussion around the mathematics curriculum, she explained that no comparison can be drawn between the achievements of children of today and children of the past: 'I think it has been dumbed down and there is no comparison with the work of 20 years ago. The standard students are achieving now is terrible in relation to ourselves' (Appendix C.1). In discussion about how she might enable children achieve a higher standard, Susan explained that to understand a 'concept more fully' children needed to 'have copies where they repeat and repeat their sums' (Appendix C.1).

4.2.3 Susan's constructivist approach to mathematical problem-solving

Following her engagement with the research project, Susan was asked to reveal how she might explain constructivism to an individual unfamiliar with the philosophy. Susan outlined: 'It is about problem-solving, finding out where the students are at and then building upon it. It's about giving a little bit more ownership to the students. It is going away from directed learning' (Appendix C.1). She added that 'it is about the children working in groups trying to reach a solution through trial and error' (Appendix C.1). When asked to describe her feelings on constructivism given that she had undertaken to become involved in

a project centred around it, Susan revealed that she felt that 'it is very valuable but it has to be used in conjunction with the rote learning, the chalk and talk and the teacher-directed learning and you can't teach your maths curriculum based solely on constructivism' (Appendix C.1).

4.2.3.1 Susan's illustration of students' experiences of learning from a constructivist perspective

When asked about constructivism from her students' perspective, what they thought of the project, Susan said that 'they enjoyed it' (Appendix C.1). To this she added: 'You will always have the one who will stop and say I can't do it, I can't do it and that causes difficulties for the rest of the group. I think though they have a very negative feeling towards maths, they have been told they are very weak by their parents, for example that things are quite serious for them going into secondary school' (Appendix C.1). In further discussion, Susan explained that mathematics from a constructivist perspective must be explored on a 'topic by topic basis' (Appendix C.1) with students who may have difficulties with mathematics: 'You would have to do it topic by topic and forget problems that require a number of concepts or operations. It would have to be simple straight forward problems' (Appendix C.1). Indeed, Susan revealed that it is not realistic to expect weaker pupils to come to an understanding of a mathematics concept or topic through experimentation and significant interaction with their peers. She gave an example: 'Even giving them a hint, you have to guide them all the way. They needed an awful lot of guidance'. But, significantly, she added: 'They still enjoyed it and did benefit from it; it made them think for a change' (Appendix C.1).

Susan revealed that she envisaged problems the children would have before engaging them in constructivism. She had to encourage her pupils constantly throughout the project, as they got to 'dead ends' (Appendix C.1) on numerous occasions. She explained that there were individuals who tended to dominate proceedings because their classmates did not know where to start and, in her words, had nothing to contribute. Susan claimed that, 'unless they (the children) are very vocal, the less able student will get lost' (Appendix C.1)

during mixed ability grouping situations. When asked if she thought approaching mathematical problem-solving from this perspective with these students from an early age would have made a difference in her opinion, Susan responded explaining:

> I think to be honest because they were particularly weak, having an idea or putting an idea about something forward would have caused difficulty in any subject area not to mind maths. They need the teacher as a crutch. They couldn't even put an argument together in English, one sentence and that was it (Appendix C.1).

However, Susan explained that it was a valuable exercise for children and she highlighted that 'children were delighted with themselves when they went some way towards achieving an answer. They might not have come up with it, but there was value in their method. It gave them a sense of positive self-esteem; it was good for them to get some sort of praise in mathematics' (Appendix C.1).

Susan explained that textbooks in use in the classroom are not sufficient in helping the teacher approach mathematics from a constructivist perspective. Specifically, the problems presented did not have enough information for the students themselves to sift through, and did not involve multiple types of operations and cross-strand knowledge. Susan felt they were 'too basic' (Appendix C.1). She sourced mathematical problems for her constructivist mathematical lessons from textbooks and the internet, in conjunction with her peers also involved in the project.

When asked if constructivist methodology would feature in her teaching of mathematical problem solving in the future, Susan indicated that it would, but that more guidance was required for every teacher. She said the 'material isn't there to facilitate the teacher' (Appendix C.1).

> It may say it in the curriculum, but I don't think many teachers would be familiar with how to go about doing it in the classroom. Class size is also an important issue; it does get loud and I have no problem with that but you can't have it at an

extreme level for a long period of time. You have to be conscious of classroom management and of other teachers and classes who might be nearby as well. Smaller numbers would be a great help (Appendix C.1).

Parental expectation is a factor in Susan's teaching.

The perception is out there that if the course or book in their opinion is not done that the children have not been taught properly in class. So the children might be going home having nothing done in their copies even though they might have had valuable discussions, and the perception might exist that there is nothing being done in the classroom (Appendix C.1).

Susan's final comment, in her final interview, was: 'I'd have to say that I would do it on a regular basis and I can see value in it. I can really see it working well with more able students' (Appendix C.1).

4.2.4 Susan's students' perspectives of their mathematics education

There were a variety of responses to this initial question.
Three students reported
- 'Feeling nervous about talking in front of people and explaining things' (Appendix C.2)
- 'Finding textbooks problems difficult' (Appendix C.2)
- 'Finding homework difficult' (Appendix C.2).

One student explained: 'I like maths because it is a challenge, and I like working out sums to get the precise answer' (Appendix C.2). This student continued to explain that it was not one of his preferred subjects but he worked at it because 'I will need it for exams in secondary school' (Appendix C.2). Another student explained: 'It is not hard to learn but if you don't learn it, it can be hard' (Appendix C.2).

When asked to describe an area of the subject that they enjoyed learning, students discussed areas of the curriculum that require the use of concrete materials. In particular, students mentioned shape and space and data: 'I like

area, drawing and working out the sides and everything' (Appendix C.2). 'I like when we measure different things in the yard' (Appendix C.2). Four students explained that they liked working out mathematical problems. One student explained that the particular problems he enjoyed involved concrete materials such as 'jugs and containers' (Appendix C.2).

4.2.4.1 Susan's students' perspectives of their mathematics lessons

Students described the teacher's teaching of mathematics from their perspective: 'We correct our homework first and then she would explain something and ask us to do questions on it. She does things on the board loads of times and then we go and do it ourselves' (Appendix C.2). 'She repeats things until everyone understands it' (Appendix C.2). Students described their teacher as one who 'explains things very well' but indicated that 'in the books, the questions are not explained very well for when we are working at home' (Appendix C.2). Children revealed that their teacher spends a significant amount of time teaching a particular concept. They explained: 'The teacher does things over and over again so it gets a bit boring' (Appendix C.2). 'We spend months at things because some people still don't get it and she stays on it' (Appendix C.2). Students named their mathematics textbook as '*Mathemagic*' and declared: 'We use it an awful lot' (Appendix C.2).

In discussion, these students explained that they have always found mathematics difficult especially problem-solving but one student revealed: 'I find it easier when we can all help each other in groups' (Appendix C.2). 'Normally, we just write down on our copies on our own' (Appendix C.2).

4.2.4.2 Susan's students' illustrations of mathematical problem-solving from a constructivist perspective

Student initial reactions to their participation in the initiative were extremely positive. They declared:

- 'They were fun to do like quizzes.' (Appendix C.2)

- 'You get to talk in class.' (Appendix C.2)
- 'They weren't all about number, some of them you had to do more than just addition, they might need addition, subtraction and division.' (Appendix C.2)
- 'I like working with other people.' (Appendix C.2)
- 'Some problems were long like stories but that made them interesting.' (Appendix C.2)

Students indicated their eagerness to work in group situations:

- 'You get to talk to your group about how you want to do it, it might be the correct answer, some people might have other ideas and it is good to see what those are.' (Appendix C.2)
- 'I like working them out with other people.' (Appendix C.2)
- 'You get to discuss with your friends and it is easier then because they help you out when you are stuck.' (Appendix C.2)
- 'If you don't understand something you can talk about it with your friends and then the whole group solves it and everyone can explain it.' (Appendix C.2)
- 'If you don't get it, some one else does and they can show you a way or you can show them the way and it makes it much easier.' (Appendix C.2)
- 'We usually do just things at our desks on our own It was our first time doing group work so it was fun just to talk about things with our own friends.' (Appendix C.2)
- 'We got on fine in our groups, we all had something to say.' (Appendix C.2)
- 'You can say more stuff to your friends than you could to the teacher. You can speak out more.' (Appendix C.2)
- 'Working together is easier and if you think about things you can figure it out before you ask the teacher all the time.' (Appendix C.2).

Students explained that they decided amongst themselves to talk about problems on the initial task. Following this, students explained they decided to look at the problem and come up with ideas that they tried out and then brought these ideas back to the group for discussion. Students explained that 'sometimes we got things wrong and we needed the teacher to explain it' (Appendix C.2). They

explained that they found the project different in that 'usually the teacher gives us as much help as you want her too' (Appendix C.2). One student declared: 'You are kind of teaching yourself how to do it rather than having someone else give it you' (Appendix C.2). When asked if they believed they could teach themselves, students revealed: 'You would eventually work it out. There are a lot of people in the groups, and eventually you would find it out' (Appendix C.2).

Students revealed that their teacher walked around looking at their work as they were engaged in problem-solving from a constructivist perspective. One student revealed: 'If you got stuck most of the time she would tell you what to do but not during this time' (Appendix C.2).

4.2.4.3 Susan's students' reflections

In discussion, students revealed the following:

- 'We learned how to work in groups.' (Appendix C.2)
- 'Teamwork is better than working alone.' (Appendix C.2)
- 'Finding different ways is good, you really understand it then.' (Appendix C.2)
- 'You learn a different way to work out a sum and you can use that in other sums.' (Appendix C.2)
- 'You are more confident and independent than before it.' (Appendix C.2)
- 'Read carefully, work it out slowly, it doesn't matter if you have to go back and start again because you don't have loads to do.' (Appendix C.2)
- 'We all helped each other and made mistakes but we tried something different because we didn't have lots to do.' (Appendix C.2)
- 'Working together is easier and if you think about things you can figure it out before you ask the teacher all the time.' (Appendix C.2)
- 'You must read the questions carefully and slowly and take your time working things out.' (Appendix C.2)

4.2.5 Susan's mathematical lessons from a constructivist perspective

The following are mathematical problem-solving lessons conducted from Susan's perception of a constructivist problem-solving lesson.

4.2.5.1 Problem 1

How many addition signs must be put between the numbers 987654321 to make a total of 99?

Student A: Ok, we have to use the big numbers to our advantage. Like 98, well we can't use 98.

Student B: You have to put in 8 addition signs because there are 9 numbers. You have to put pluses between each one.

Student C: Yeah, nine numbers so 8 addition signs

Student D: Yes 8 addition signs

Teacher: But would they add up to 99 if you used 8 addition signs. 9 + 8 is 17 plus 7 is 24 plus 6 is 30 plus 5 is 35 plus 4 is 39 + 3 is 42 plus 2 is 44 and plus 1 is 45. So no, try and put some of the numbers together.

Student B: What about 1 + 2 + 4?

Student C: But it will still give you 45.

Student A: We have to be careful of how we use the big numbers.

Teacher: How about joining your 7 and 6 together maybe?

Student A: You put the 2 and the 1 together that's 21 and 9 is 30.

Student B: There are lots of ways the 3 and the 5 together is 35.

Student A: We have to be careful and keep the 9 and the 8 separate – careful how we use the big numbers.

Student B: Are we allowed move around the numbers or do they have to be like that?

Teacher: Start at the beginning and work it out.

Student B: Are we allowed use 7 and 2 for example?

Student C: 54 + 9 +

Student A: 76 + 8 +

Student B: There are lots of ways. Lets try lots and see what happens.

Student A: You need 91 is it?

Teacher: 99

Student B: I have it, 65 plus all those numbers.

Student B: Or 43 + 21 and then add all them on.

Student A and C: Yeah, yeah, 43 + 21 and then add all them on.

Student A and C: Teacher we have it.

Student A and C: It's 65 + 9 +8+7+4+3+2+1.

The teacher concludes by using the students' example and showing it to other students by modelling the solution on the blackboard. The students' finished example is displayed below.

Figure 6: Susan's students' work (Problem 1)
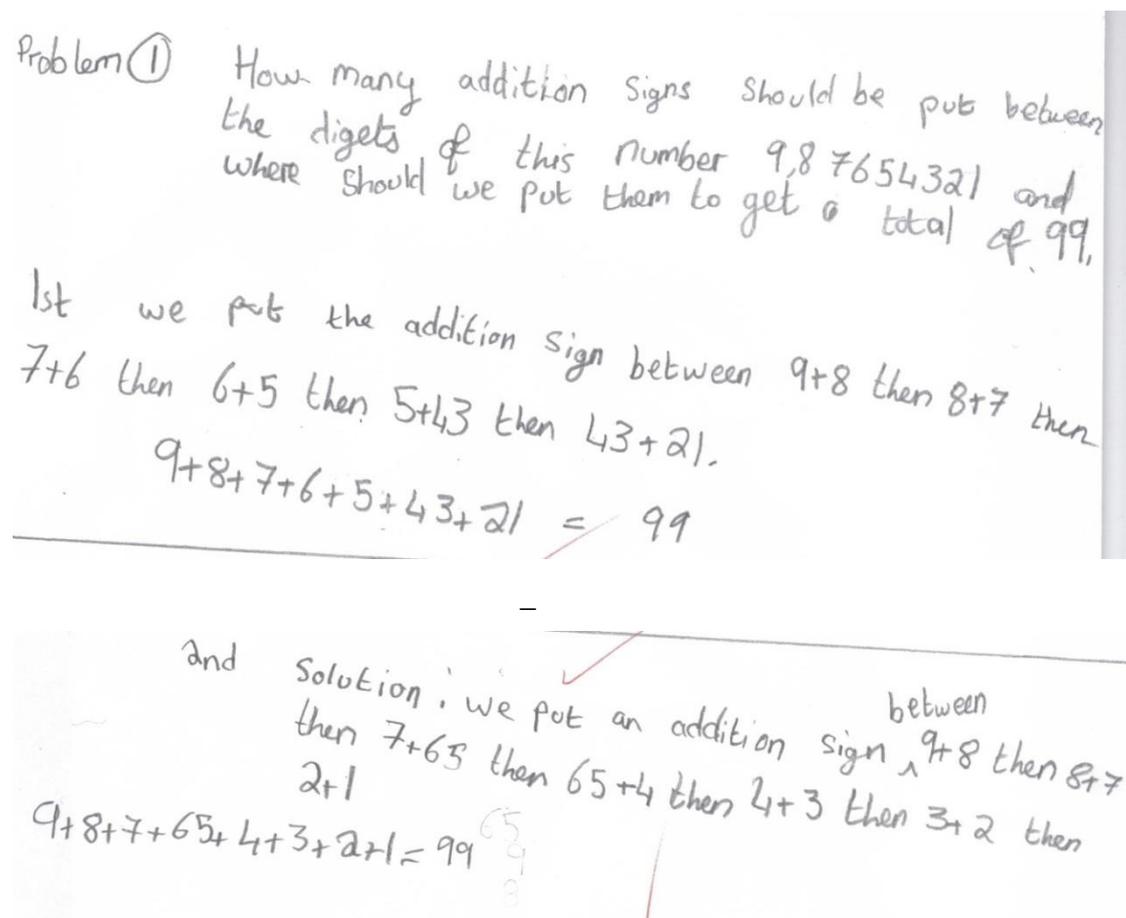
Susan placed emphasis on student interaction in her teaching episodes. On this occasion, she interacted with the students frequently in their attempts to solve the problem. Susan restricted the students' discussion in the initial phase of the

problem-solving procedure by encouraging the students to 'put some of the numbers together'. Susan directed students towards appropriate strategies for solving the problem rather than allowing them to develop their own personal strategies, for example: 'How about joining your 7 and 6 together maybe?' This information was critical in enabling students to develop a method to solve the problem. The students arrived at two solutions to this mathematical problem and these are displayed above. The teacher concluded the lesson by teaching the rest of the class the solution found by the students.

4.2.5.2 Problem 2

A farmer has pigs and chickens. She counted 140 eyes and 200 legs. How many pigs and how many chickens were there?

Student A: So you divide 140 by 2.

Student B: Why?

Student A: Because of eyes, they all have 2 eyes.

Student B: So there will be 70 eyes then?

Teacher: Good start, but not 70 eyes there are 70 …

Student B: Pigs?

Student A: Chickens?

Teacher: No, you were right with your eyes, you were right to divide by 2, so 70 …

Student A: Animals, because there are 70 pigs and 70 chickens

Student B: 140 means altogether as there are 70 pigs and 70 chickens.

Student A: Wait, maybe we should divide 70 by 2.

Student B: If we divide that 140 by 2 we are getting 70, where is that other 70 gone?

Student A: Yes but that's how many animals there are now – 70 each has 2.

Student C: Let me think for a second.

Student C: Let me divide 70 by 2, that is 35 so.

Student B: If we divide 70 by 2 we find how the pigs' eyes have… how much eyes the pigs have.

Student B: 70 – we are right the way we are – it's 70 pairs of eyes.

Student A: This is complicated we need the teacher.

Teacher: Can I give you a hint, some people have worked out that if there are 140 eyes in total, and there are 70 animals altogether as each animal has 2 eyes.

Students A, B and C: Oh, 70 animals.

Teacher: So now we have to figure out all the different ways of making 70 and see which would make sense. Take a guess, 30 chickens, so 2 legs each is 60 legs and then there would be 40 pigs and 4 by 40 is 160 – so it is 230, could that be right?

Class: No!

Teacher: It's all trial and error – that's what we have to do make guesses and check them out.

Teacher: Don't rub out any of your answers. Remember the eyes are sorted and that it is the legs that we need to work on. Have we an answer?

Student A: 30 pigs and 40 chickens?

30 pigs and 40 chickens have 140 eyes so that is right.

30 pigs will have 120 legs and 40 chickens will have 80 legs.

Teacher: That is 200 legs altogether, that's right, well done.

Students' work is presented below.

Figure 7: Susan's students' work (Problem 2)

Students were very confused when presented with this problem. This confusion is evident in their written work. Students failed to read the problem carefully and think about it. Susan did not encourage students to recall the procedure they agreed upon for engaging in mathematical problem-solving. Students became focused on operations at the beginning of the problem and failed to revisit the problem and consider the information contained within. For example: 'We divide that 140 by 2 we are getting 70, where is that other 70 gone'. Rather than facilitate their discussion, the teacher provided the children with hints: 'Can I give you a hint, some people have worked out that if there are 140 eyes in total, there are 70 animals altogether as each animal has 2 eyes'. Susan went on to teach the children the solution to solving the problem: 'So now we have to figure out all the different ways of making 70 and see which would make sense. Take a guess, 30 chickens, so 2 legs each is 60 legs and then there would be 40 pigs and 4 by 40 is 160 – so it is 230, could that be right?' This provided the stimulus for one student to answer the problem correctly. This solution is presented in the bottom left hand corner of figure 3.

4.2.5.3 Problem 3

In how many different ways can the carriages of a three car train be arranged?

Student B: Let's pretend that these are them a grey one, a white one and a red one.

Student A: Are these trains?

Student B: Carriages

Student C: Let's just do this for a minute; don't worry about the drawing, that doesn't matter.

Student A: We have colours, that makes it easier to see.

Student B: We draw them all out. Put the white one first, the grey one last and the red one in the middle.

Student C: Now you can mix them up?

Student B: Red first, grey last and white one in the middle

Student A: So 4 times

Student C: No you can do it once more – Teacher it's 4 times.

Teacher: The group over here have 6 – anyone have a different answer?

Student A and B: How could they get 6 ways?

Student A: Do red one first, grey one last and white one in the middle, have we that one?

Teacher asks another group to stand in front of the class and explain their answers.

Teacher: Student X – will you explain to us please what you did.

Student X: I can't really remember by looking at this.

Teacher: What Student X is trying to say to us is that her group named the carriages 1, 2 and 3. You could get 1, 2, 3 you could get 1, 3, 2. Then you might put number 2 first and get 2,1,3 2,3,1 3,1,2, and 3,2,1. They are all the different ways they can be arranged so let's count them – 6. I think most groups got that, Good job, well done.

It is clear that students are not in the habit of reflecting upon the problem-solving situation and engaging in discussion around the problem before

proceeding to adopt a strategy for solving the problem. However, students structured a good method in their initial attempt to solving this problem. It is clear from the above that students were capable of solving this problem using a variety of methods. Students A, B and C chose to represent the cars of the train using colour. A representative of another group, Student X, decided to label all cars using the numbers 1, 2 and 3 and randomly order the cars of the train as illustrated above. Susan chose to interrupt pupils' attempts at solving this problem and utilise the solution designed by Student X's group to illustrate an answer to this problem to the rest of the class. This is significant, as the particular group under observation had identified an appropriate method of solving the problem and were on the correct path towards a result yet the teacher chose to interrupt their activity abruptly.

4.3 Participant two: Emily

Emily was a participating teacher teaching in a Limerick city school. The following is a photograph of Emily's classroom.

4.3.1 Emily's profile

Emily has great experience of teaching at primary level. She is a fifth class primary teacher and has been teaching either fifth or sixth class for the past thirty years. Emily is a fully qualified National Teacher. Emily admitted feeling nervous about undertaking this project but from the outset was very interested and open to exploring mathematical problem-solving from a constructivist perspective. Emily revealed that she has a genuine love for the teaching of Irish to primary pupils and that the teaching of mathematics would come second to that. She highlighted that this could be an explanation for her initial nervousness. Emily explained that it was only when the lessons were put into practice that she understood the principles behind constructivism and the objectives of the lessons. Emily revealed: 'I was a bit hesitant at the start because I know you are interested in maths and I'm interested in Gaeilge and I didn't think I'd be able for it but once you showed me that it is the solving of problems and the methods children employ to get there it was a really enjoyable and interesting experience for me and for the children' (Appendix D.1). Emily

has high expectations of all the students in her class because, she explained, 'I have high expectations myself' (Appendix D.1). Initially Emily was hesitant in exploring problem-solving with her students from a constructivist perspective and, therefore, requested the researcher to visit her classroom and conduct such a lesson. The researcher duly obliged by visiting her classroom and conducting a mathematical problem solving lesson from a constructivist perspective utilising the four stages of Polya's (1945) heuristic.

4.3.2 Emily's teaching of mathematics

Emily studied ordinary level mathematics to Leaving Certificate level. She believes the enthusiasm and interest shown by teachers of mathematics at second level was a factor in her choosing to study higher level mathematics to Intermediate Certificate level and ordinary level mathematics to Leaving Certificate level.

> In secondary school it kind of depended on the teacher that you had. I was, I believe, an average student at maths but then we went through a couple of years at secondary school where the teacher was not that great herself and therefore didn't give us any real love of the subject. It depended on who was teaching you. I did honours maths to Junior Cert and then pass as I felt that I didn't have the grounding (Appendix D.1).

At primary level, Emily admitted that corporal punishment was part of the routine of the classroom and that it had a place in the teaching and learning of mathematics during her days at school. She revealed her interest in and passion for mental mathematics stems from her own days at primary school.

The emphasis in those days was on mental maths and he (the teacher) was very interested in doing lots of mental maths problems. We were very fast thinkers. That was also the time though of the bata and, on a Friday morning we would have a special mental maths competition whereby he would have all of both fifth and sixth class by the wall with the bata balanced on his middle finger under his coat and depending on

how fast or how slow you were, you didn't get the bata. It made you think pretty quickly (Appendix D.1).

Emily could not recall in detail her experiences of learning to teach mathematics during her time at third level. However, Emily did explain that 'we did nice things like bar graphs nice airy fairy things, I can't remember the basics though' (Appendix D.1). There was significant emphasis placed on the learning of mathematics tables during Emily's own primary school year: 'In my day we could do our tables, we knew them absolutely inside and out' (Appendix D.1).

Emily places significant importance in the exploration of the number strand of the Primary Mathematics Curriculum (1999) explaining

> I'm old stock, I like to stick with the nitty gritty number, fractions, decimals, percentages and the like then move on to the more light-hearted areas as I like to call them of data and chance and length. I love the end of the year when I leave it for algebra. They do get a good grounding in the room but the focus is on number. I give 60:40 to the number strand compared to everything else (Appendix D.1).

Emily admitted sticking to 'tried and tested' (Appendix D.1) methods of teaching mathematics that she believes work for her. It is clear from Emily's classroom and the daily work of her pupils that she places significant emphasis on rote memorisation. Emily displays all of the multiplication and division facts in clear view of the students and the students have 'all multiples to fifteen written in their copies numerous times, so now that when we are doing fractions, they are able to pick lowest common multiples when it comes to simplification. They need this' (Appendix D.1). Emily believes her teaching of mathematics has become 'more structured' (Appendix D.1) since leaving college. When asked whether or not anything influenced her teaching of mathematics she explained that in the past, entrance examinations would have influenced her but nothing more than her own high expectations.

Emily describes a good mathematics student as one who 'can do any calculation involving simple numbers to fractions' (Appendix D.1). She explains that 'any child should be able to do a sum in black and white in front of them, the ones with the signs between the numbers' (Appendix D.1). Furthermore, she added: 'A good student should be able to read a problem, see what is being asked and then find a step by step approach to the solution and then work it out accurately' (Appendix D.1). Weaker pupils, Emily added, 'will forget the topic within a week after finishing' (Appendix D.1).

In conversation about the Primary School Curriculum (Government of Ireland, 1999a;1999b) in-service education that teachers received, Emily explained that she found it 'informative' (Appendix D.1) and realised 'maths can be made so interesting for children' (Appendix D.1). She highlighted the fact that the curriculum places great pressure on teachers and that she tends to experiment with teaching methods encouraged for the exploration of 'data and chance' (Appendix D.1). Again Emily repeated she likes to stick to an 'old fashioned' (Appendix D.1) methodologies in her teaching of mathematics. She explained 'I branch out every now and then into areas of maths that I would not be too sure of myself' (Appendix D.1).

Emily does use pair work and group work at times but emphasised her need to teach in a very structured environment. Emily has the students in her classroom correct each other's work explaining: 'It makes children more aware of pitfalls, I believe when they are doing their own sums' (Appendix D.1). Emily regularly calls on the 'confident children (Appendix D.1) to come to the blackboard and do some examples of work. These are students, Emily explains, she knows will succeed.

4.3.3 Emily's constructivist approach to mathematical problem-solving

Prior to discussing a constructivist approach to mathematical problem solving, Emily was asked if she had ever tried anything innovative in her teaching of mathematics. She replied: 'I would have to say no, but I was fascinated by this project' (Appendix D.1).

In discussion around the core principles of the primary curriculum Emily gave her interpretation of them: 'Well, it's about taking the maths out of the room, integrating with other areas. It is, I suppose, child centred and that is more appropriate' (Appendix D.1). Emily was then afforded the opportunity to describe constructivism to an individual who may not be familiar with it. She explained that 'it is putting an interesting task on paper in front of children and getting them in a group and trying to solve a problem. It's not showing them how to get to the answer but directing them if needed. Popping questions out there to make them think in the right direction is useful' (Appendix D.1).

This led to initial conversation about the project all participants agreed to engage in. Emily was asked to describe her general feelings about including constructivist methodology in the curriculum. She said

> I find it fascinating actually. It has its place but it wouldn't be a major theory in my view. It could not come before basic work. The class were pretty good, there were the individuals who were extremely good and the mixed ability groups worked extremely well. The good individuals in a group were able to bring other students along with them. It would be a very good stepping stone to solving problems yourself on your own (Appendix D.1).

Emily spoke at length about her initial experiences in the classroom when engaging with mathematical problem-solving from a constructivist perspective. She acknowledged that the whole experience was new to her and that she herself learned as the lessons progressed.

> We got hooked on the problems; I have to say if I hadn't a high level of achievers and had an average class it might not work as well as it did. I do intend to include them though in all classes. We have added Sudoku's to the lessons that we still do and they enjoy solving those in their groups like the problems also. I am learning with the children as it is so new. When we are all learning like that including me it brings excitement to the room and they want more of it, it is great (Appendix D.1).

It is clear that Emily places great value in approaching mathematical problem solving from a constructivist perspective but, she continuously cited the

pressures of curriculum and monthly reports as inhibiting factors to her use of this approach on a regular basis. In an evaluation of trial and error, Emily suggested:

> It could be used at some time during the year but you are under such pressure curriculum wise for every sort of subject. I'm the type of person that will relax and try that when I know that I have done everything else. So if it comes to after Easter time and I have my entire maths curriculum done I might say 'right guys, lets give this a try'. I can relax then because my work is done. With the monthly scheme it's hard to find time (Appendix D.1).

Emily went on to give a description of how her day-to-day teaching:

> On a day-to-day basis the time isn't there are too many demands and pressures. In sixth class in particular you are inclined to go for Irish, English and Maths and even English, Maths and Irish, it is coming to that order now and then hopefully get around to the religion programme with all Masses, etc. after that and then squeeze in the rest. What I'm finding is I get the major four out of the way and then spend a day or two doing the other subjects like history, geography en block. Then I'm happy that I have my work done (Appendix D.1).

From the outset, Emily was willing to engage thoroughly with this project. She displayed enthusiasm at both the planning and implementation stages. Emily indicated early on in the term that she would require the researcher to come to her classroom and give a demonstration of constructivist teaching. Emily said: 'You could have talked forever before you came into the classroom and I would not have understood what you were at' (Appendix D.1). Emily's diligence in her approach to teaching was reflected in her extreme interest and consistent note taking as the researcher was in the classroom. Following this episode, Emily commented: 'I found the very first day that you took the class and guided me in taking future ones was invaluable. It motivated me to a significant extent. I need to see things in action to understand. It would be invaluable for student teachers to observe' (Appendix D.1).

4.3.3.1 Emily's illustration of students' experiences of learning from a constructivist perspective

Students in Emily's classroom were highly motivated by mathematical problem solving from a constructivist perspective as illustrated by the following.

> They were totally motivated by it. They were not happy that it was run for just a defined period of time, they wanted to do it all the time. They might just want to get away from the more structured lessons that they were normally used to. We had a standing up row one day. My father got one answer and he got another answer and it caused great debate altogether. They made me do it again, we had it on paper and we acted it out and we came to the conclusion that we were wrong and my father (teacher's father) was right. It was great, it was very motivating (Appendix D.1).

The organisation of the problem-solving lessons proved of no difficulty to Emily. She found it easy to assemble them into groups and distribute any materials or resources that they might have required. Emily found it 'amazing' (Appendix D.1) that different groups of students in the classroom could come up with a number of different ways to solve a single problem. She admitted that groups of students came up with solutions to mathematical problems that she would not have even thought of herself. Emily talked about how she dealt with difficulties children may have experienced as they solved problems chosen for them by the teacher. She explained that if children could not see a pathway to a solution they needed to be given a prompt to direct them, or their interest in the work would be quickly lost.

Emily struggled personally in dealing with students, finding participating in the group problem solving exercises challenging. She revealed that she felt 'wary' about those children for whom the path to a solution was not clear: 'I kind of felt very upset for them, they were missing out on something special. They ended up on the periphery and ended up letting everybody else do the work. They were not getting involved themselves really, it was a disadvantage for them' (Appendix D.1). Emily was unsure if these students benefitted from the

experience, explaining that she did not believe they had the 'mental capacity' for the work involved. When asked if she believed they were helped in any way by other members of their group, Emily explained that, as they were a very friendly class, 'they did their best to include … but I could see he was totally lost' (Appendix D.1). Emily suggested grouping the students according to ability and giving them 'extremely simple' (Appendix D.1) problems with a lot of teacher guidance. She suggested problems that included visual images and clues might be more appropriate that those involving the written word.

4.3.4 Emily's students' perspectives of their mathematics education

Some children explained that mathematics was not one of their favourite subjects. Their comments included:

- 'Its not one of my favourite subjects.' (Appendix D.2)
- 'Scared in case you get it wrong.' (Appendix D.2)
- 'Sometimes it's easy but other times it can be hard. Some of the problems can sometimes be fun but sometimes they can be boring.'(Appendix D.2)
- 'I get scared sometimes as I can't do things.' (Appendix D.2).

Students discussed their favourite aspect of the subject. A student explained that he liked it and specifically mentioned 'numbers and working with fractions' (Appendix D.2). In further discussion students revealed having difficulty with problem-solving: 'When you get stuck on your own with things it can be annoying' (Appendix D.2). 'Sometimes lots of long division or when you get stuck on a problem and you are there for ages working on it' (Appendix D.2). Another student explained: 'I feel nervous when I get something wrong because you might be shouted at or yelled at for it' (Appendix D.2). 'Division, we do lots of it and I don't like it at all. If you get it wrong you get in trouble. I still don't get it at all' (Appendix D.2).

Students were asked to reveal areas of mathematics that they like to study, or describe some mathematics lessons they had really enjoyed in the past. All

students indicated their interest and delight in working with concrete materials and interacting with the local and school environments. Students revealed:

- 'I like doing length, charts and things.' (Appendix D.2)
- 'Data, all of it, surveys and things like that.' (Appendix D.2)
- 'I like things that are different, where I'm out of my place.' (Appendix D.2)
- 'Length, we had measuring sticks.' (Appendix D.2)

One student explained that during their time in fourth class the teacher played many maths games with them and that there was always 'excitement' (Appendix D.2) in the class during this period. One student, who explained he liked mathematics revealed 'division is easy and addition too' (Appendix D.2).

4.3.4.1 Emily's students' perspectives of their mathematics lessons

Students described the teacher's exploration of mathematics from their perspective. From the following, the students' daily mathematical experiences are largely based based on individual textbook activity: 'We have places, maths places and English places by the order of the Drumcondra, the best are at the back' (Appendix D.2). 'We generally work on our own, sometimes we compare answers but usually we just work in pairs ourselves' (Appendix D.2). 'We start with mental maths, 20 questions and we have 5 minutes to do them' (Appendix D.2). 'Then we move on to our *Mathemagic* for ages' (Appendix D.2). 'We work on our own most of the time except when we have to use things like the measuring sticks we had one time' (Appendix D.2). 'We never worked in groups for maths until now' (Appendix D.2). 'We do one topic at a time like long division or time or something' (Appendix D.2).

4.3.4.2 Emily's students' illustrations of mathematical problem-solving from a constructivist perspective

Student reactions to their participation in mathematics lessons from a constructivist perspective were positive. One student explained: 'They were hard because they were longer than usual. They could have been a lot shorter. They could have been less confusing but they did make you think though' (Appendix D.2). Another continued: 'After a while we learned to read and to look for what we needed' (Appendix D.2). Students were positive about interacting with each other during problem-solving activity: 'I like doing it in groups because some know the answers to some and others know the answers to others so we could help each other out' (Appendix D.2). 'They were really fun because we got time to work together. If you are stuck, there is always someone else there to help out as well' (Appendix D.2). Students explained that the problems were 'more interesting than normal ones to figure out' (Appendix D.2).

Students were asked to describe in detail what they do in their attempt to solve a mathematical problem following their participation in the project. They explained: 'We go through it for a plan, and then talk about what we think about it before we go and do anything' (Appendix D.2). 'We read the problem, read over it again and then made a plan in our groups about how we did the problems' (Appendix D.2). 'We would talk about what is useful and what isn't' (Appendix D.2). 'We have a highlighter for the important things so you don't have to read over the whole problem again' (Appendix D.2). 'We write down what comes into our heads on paper. Sometimes we just talk and we don't write anything' (Appendix D.2). 'Sometimes we all did them ourselves and then if any people got the same answers we would come together and see who had the same answers and then talk about them so that we could see that we were right' (Appendix D.2). 'Everyone had different ways and some ways might have been more easier to understand than your own ones and we could plan using the easiest way then' (Appendix D.2). 'You really know how to explain it to someone afterwards because we might have different ways of solving the same problem' (Appendix D.2). 'Explaining something out loud helps you

understand' (Appendix D.2). 'We used diagrams and materials that we would not normally use' (Appendix G.3).

Students explained that the teacher would often tell the children they were either correct or incorrect in their calculations or tell them: 'You are almost there' (Appendix D.2).

When questioned about solving problems that they might not have been familiar with, students declared that it is helpful for everyone in the group to record their thoughts on the problems before progressing to the planning stage. Students highlighted the need to 'break it all down so that you can do it in little steps' (Appendix D.2). Students explained that, although they may be more difficult than the problems assigned to them in an everyday mathematics class, 'they were harder in a good way' (Appendix D.2). One student revealed: 'I learned that if something is long and confusing, you can break it all down so that you can do it in little steps' (Appendix D.2). 'In the end, it took a little longer, that's all' (Appendix D.2).

4.3.4.3 Emily's students' reflections

In discussion, students revealed the following:
- 'They were more fun, it was kind of like doing projects and not maths at all.' (Appendix D.2)
- 'I thought it would be hard but because they (the problems) were interesting, you kind of stayed at them in the group.' (Appendix D.2)
- 'Don't always think that you are right, discuss and listen to other people's opinion.' (Appendix D.2)
- 'It's good to work as a team. Check and recheck your answers. There is no pressure because you are not racing to get to the end of a page.' (Appendix D.2)
- 'I thought they would be hard and I was scared at the start but they were fine and it was easy when we made up plans and everything.' (Appendix D.2)

- 'You could do it in groups and that was more fun. It wasn't about all getting the top marks in the room, it was just about the one question at the time.' (Appendix D.2)

- 'I was more about how you got the answer than the answer, how you worked it out.' (Appendix D.2)

- 'There can be lots of different things in the problems not just one like addition. There can be division, subtraction and multiplication all in the one sum.' (Appendix D.2)

- 'Sometimes you are nervous though if you are one of the smartest in the group because then other people are relying on you.' (Appendix D.2).

4.3.5 Emily's mathematical lessons from a constructivist perspective

The following are mathematical problem-olving lessons conducted from Emily's perception of a constructivist problem-solving lesson.

4.3.5.1 Problem 1

A hare and a tortoise are 40 km apart. A hare travels at 9km an hour and the tortoise at 1 km per hour. How long will it take them to meet?

Teacher: How long will it take the hare to get to town travelling at 9 km per hour and it is 40 km from Haretown to Tortoiseville? Make you sure you talk out loud throughout. Remember also what you have to do before problem-solving.

Student A: About 4 hrs.

Teacher: Why do you think 4 hours? Remember he travels at 9 km per hour.

Student A: 4 hrs to get 36km but he is still not there.

Teacher: So if he goes at 9km per hour, how long will it take to go 4km (Repeated).

Student B: About half an hour or 25 minutes

Student C: Yes it takes less if he can do 9 in a full hour.

Teacher: Ok, now turn your attention to the tortoise.

Student A: He can go 1 km an hour.

Student C explains:

I've drawn 40 bars alongside the page. Each bar is like a km. So if we draw after an hour the hare will have covered 9 of those so the hare 1 2 3 4 5 6 7 8 9 and at the other side the tortoise would have covered 1. Now the hare will travel another 9 so 1 2 3 4 5 6 7 8 9 and that's 2 hours. The tortoise will go 1. Now the hare will travel another 9 so 1 2 3 4 5 6 7 8 9 and that's 3 hours and the tortoise goes another 1.

Figure 8: Emily's students' work (Problem 1)
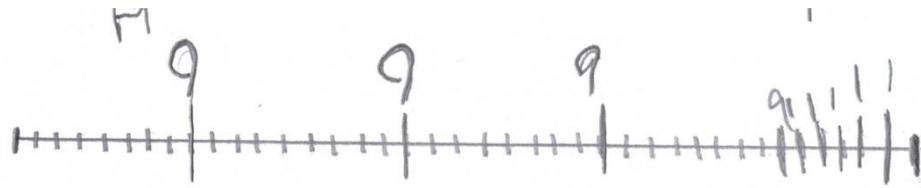
Student B: Then the tortoise goes another 9, 1 2 3 4 5 6 7 8 9 and the tortoise goes 1 so that is 4 hrs. 4 hrs, we have the answer 4 hrs.

Student A: Look, compared to the tortoise, the hare actually travels quite fast.

Emily began this problem-solving session by restating the problem for the children. At this stage Emily was providing the children with considerable insight into how the problem should be solved. This problem proved to be easily solved as the teacher encouraged students to focus on the relevant information contained in the problem statement. Students did not spend a significant amount of time devising a strategy to solve the problem or in conversation about a method for solving the problem. Emily repeated an instruction early on in the solving of the problem: 'So if he goes 9 km per hour, how long will it take to go 4 km'? In repeating this statement, Emily stressed the 9 km per hour and 4 km to ensure children made the connection between both factors.

4.3.5.2 Problem 2

Emily encouraged students to write an explanation of how they solved the problem on this occasion. They were asked to detail their solution to solving the problem but also include other methods they employed that did not succeed.

Jane received a new doll from Aunt Maggie as a present for her ninth birthday. She also received a package containing a variety of clothing for the doll. The package contained a red hat, a blue hat and a green hat. It contained a yellow jumper, a brown jumper and a purple jumper together with a pair of white socks, a pair of blue socks and a pair of navy socks. Aunt Maggie was interested to find out how many ways Jane could dress the doll. How many ways could Jane dress the doll?

Teacher: Read the problem and decide as a team what to do.
Student A: I think we could just draw stick men and put different clothes on them.
Student B: Yeah, we can use colours. I'll get them. We can do it like this then.

Figure 9: Emily's students' work (Problem 2)

Student A: That's going to take ages. Let's go so and we could take turns using different colours, I'll use the red hat and you can use the other ones.
Student C: What about, first all of our nine men with the red hat?

Figure 10: Emily's students' work (Problem 2)
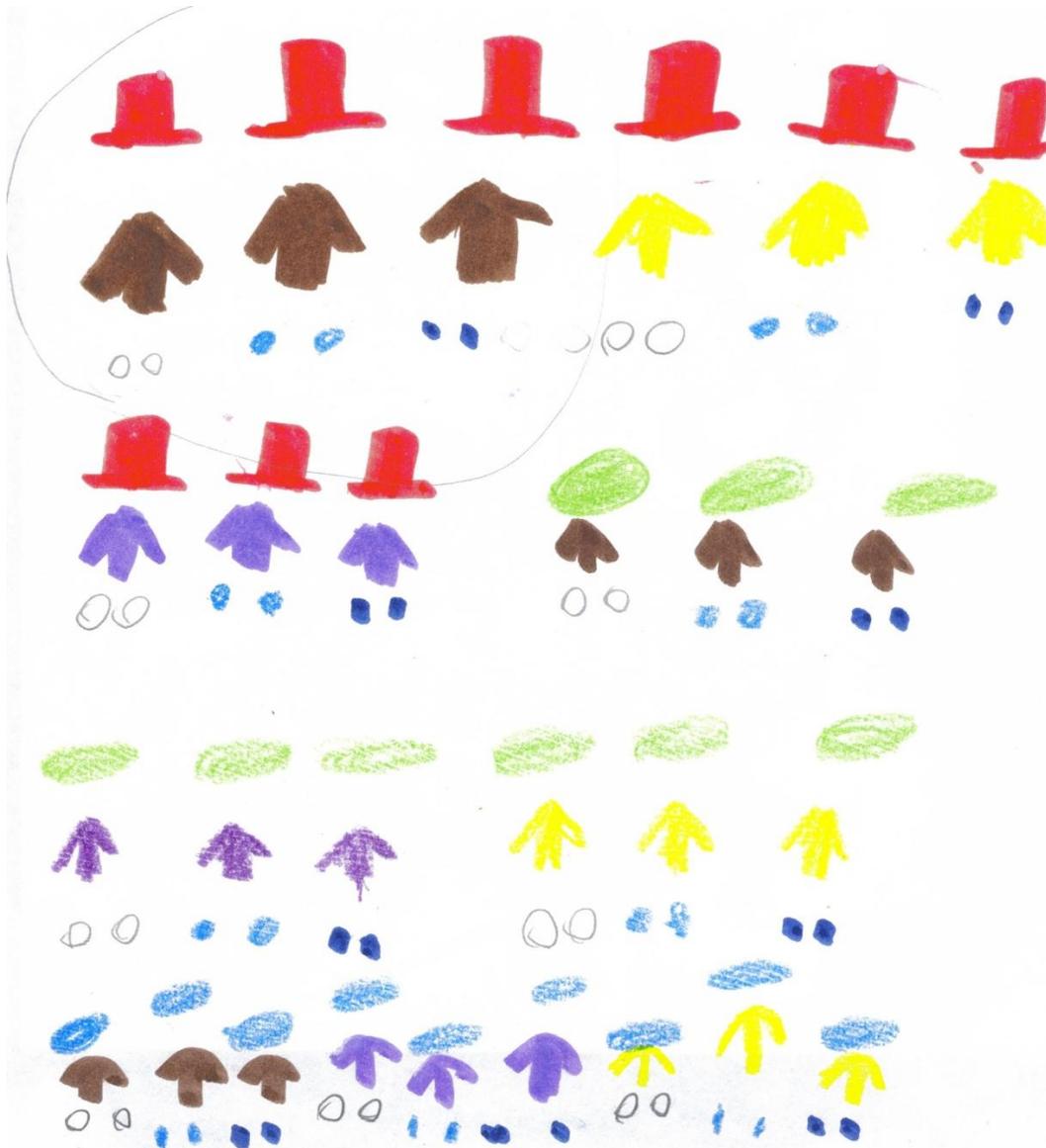
Teacher: What did ye do?

Student A, B and C: We tried to do all the ways we could using the red hat and got nine ways

Student A, B and C: With the blue and the green hat it will be the same so we got nine times three and got our answer – 27.

Student A, B and C: The other way we tried to do it was by drawing lots of stick men and start putting anything on them.

Students developed a very sophisticated method for solving this problem very quickly after finding out that their original idea for a solution was 'going to take ages'. Students worked methodically with the colours to identify an answer to the problem that was very appropriate. Students realised that the initial strategy that they had chosen to solve the problem was inappropriate and had no difficulty in revisiting the strategy to modify it. There was little teacher interference in the problem-solving activities of the students with the exception of the teacher asking the students to explain their chosen method of solution.

4.3.5.3 Problem 3

King Arnold sits at a Round Table. There are three empty seats. How many ways can 3 knights sit in them?

Teacher: Firstly, think, have I done something like this before, is there a method I have used before that might be useful?

Student A and B: Lets draw a round table with chairs, 4 chairs, empty ones.

Student A: Ok now what do we do?

Student B: Let's read it again aloud together.

Student A: 3 empty seats so and King Arnold in one of them

Student B: Let's draw him.

Student C: What do they mean about 'how many different ways?'

Teacher: Can anyone tell me quickly the important information and maybe also tell me some information that is not important.

Student C: The name of the King

Student B: The round table and the 4 chairs are important.

Student A, B and C: This is very hard, let's read it again and again.

The Students read the problem

Student B: Maybe it is kind of like the doll problem − one of them might sit here then move to here and then to here.

Student C: And then, that one swaps and sits in the other seat.

Student A: So draw one table and draw a crown at the top. Then we will call them King 1, King 2 and King 3.

Student B: Why King 1?

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Student A: Ok, Knight 1, Knight 2 and Knight 3.

Student C: Yes, Knight 1 can be pink, Knight 2 can be blue and Knight 3 can be green. Now we will move them around.

Student A: Yeah, now Knight 1 moves to Knight 2 seat, Knight 2 to Knight 3 seat and then Knight 3 moves to Knight 1 seat.

Student C: So that is nine. Is it?

Teacher: Can you explain for me your answer?

Student B: We drew a round table and the king was on top and his crown was in yellow. Then we had King 1 at the first chair, K2 at the second chair and K3 at the third chair.

Student B: Then we moved K1 to K2 chair then K2 to K3 then K3 to K1 chair.

Teacher: Good

Student C: Then we did it all over again

K1 went to K3.

K3 went to K2.

K2 went to K1.

Teacher: Make sure you write an explanation.

Student A: For the second part of the sum we drew another round table. This time there was 4 empty seats.

Figure 11: Emily's students' work (Problem 3)
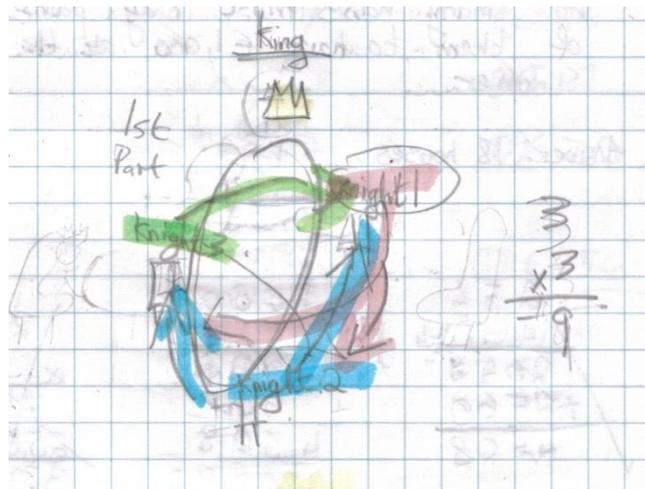
Emily facilitated the students in their problem solving endeavours by asking purposeful questions throughout the teaching episodes.   These questions included: 'Firstly, think, have I done something like this before, is there a

method I have used before that might be useful?' and also 'Can anyone tell me quickly the important information and maybe also tell me some information that is not important?'

Emily followed in detail Polya's (1945) four-stage problem-solving procedure and this is clear from the above example. Students engaged with coming to an understanding of the problem as the teacher encouraged students to recall a previous similar problem. Students devised a plan for solving the problem and proceeded to carry out this plan. Students decided to draw an illustration of the situation, rename the knights, and solve the problem. Students developed a sophisticated problem-solving strategy recalling a similar problem solved previously. Students chose to draw diagrams during the initial stages of the problem, renaming the knights K1, K2 and K3. Students reflected on the problem-solving situation and Emily encouraged students to explain their answers.

4.4 Participant three: Joe

Joe is a participating teacher in a Limerick city school. The following is a photograph of Joe's classroom.

Photograph 3: Joe's classroom

4.4.1 Joe's profile

Joe has been teaching at primary level for thirty years. He has an undergraduate degree in education and has taught at first, fifth and sixth class levels. Joe prefers teaching at senior class level. Mathematics is a subject Joe likes to teach.

> It is a favourite subject of mine because when I was at school myself I really enjoyed maths and then when I got interested in it, I thought it was an area I should strive to teach well. I always enjoy doing it during the day, I don't mind when maths time comes around. I like it, there is plenty of variety and challenge in the subject (Appendix E.1).

He stated that he has a particular flair for the subject and that he is, therefore, eager that students would grow to enjoy the study of mathematics also. He explained:

No matter what faculty they go into, they need a good knowledge of the maths of anything. If they have that they are at an advantage already. It can build confidence; they can take on problems and deal with them. It gives them different opportunities and opens doors for them (Appendix E.1).

Joe was very interested and eager to be involved in the project, as he indicated: 'I like plenty of variety and challenge in the subject' (Appendix E.1). Joe was very open to engaging in constructive discussion around mathematical problem-solving and the more appropriate ways to approach it with students at sixth class level. Since beginning his career in teaching thirty years ago, Joe revealed the biggest change he has witnessed is the introduction of an 'easier' (Appendix E.1) curriculum.

4.4.2 Joe's teaching of mathematics

Joe was initially asked to describe the principles on which the Primary Mathematics Curriculum is built. He said:

I suppose it is very much focussed on the full participation of the child. That they engage with the topic fully and that they understand what they are at. We are not to present them with abstract concepts anymore as we might have done in the past. It's child-centred. It constantly emphasises that we must use concrete materials. I think that is very important (Appendix E.1).

Joe believes teachers became complacent in the past and moved away from work, such as the above, that mattered. He explained that a focus on computation resulted in children having little or no understanding of the actual concept behind the mathematics. He maintained material was 'learned by rote and of no use, unlike carrying something out with concrete materials' (Appendix E.1).

Joe believes it gives the 'average' (Appendix E.1) or 'weaker' (Appendix E.1) student time to improve, as it is more child-friendly. He said: 'The problems and topics are not as difficult as they were, for example, 10 or 15 years ago' (Appendix E.1). It is necessary, therefore, in Joe's opinion, for a teacher to be

able go beyond the curriculum at present. Joe revealed that some students will find the mathematics curriculum less than challenging and that it is necessary to have the ability to plan for lessons that go outside of the confines of the curriculum and tend to cater for these pupils' needs: 'You can challenge a good class if you put your mind to it. There is no doubt that the programme has been watered down a lot' (Appendix E.1).

Joe explained that a good mathematics student is one who can deal with a wide range of mathematical problems and work out appropriate strategies to solve them: 'Some children are good to reason, to get from the known to the unknown. That's nice to see' (Appendix E.1). To give the students practice of this, Joe presents them with different sources of information to solve a singular problem. He emphasises the importance of giving students problems that 'draw on their knowledge of various concepts' (Appendix E.1), so that children 'use bits of knowledge simultaneously' (Appendix E.1). Joe frequently sources mathematical problems outside of the traditional textbook.

> I have old ones from the past and some of my own examples gathered from newspapers and the like over the years. It takes time but it is beneficial. If you really want to challenge those who achieve you have to go far beyond the normal day-to day work. That's when you see the students have potential really (Appendix E.1).

Joe is very interested in pupils' mathematical problem-solving ability. He explains that a student is a competent mathematical problem solver when 'you give a student a problem involving a number of different operations or concepts and they can solve it easily. If they are quick and transfer and use knowledge in different situations, you can be sure they have a good understanding' (Appendix E.1).

Joe is eager to incorporate fresh approaches to his teaching of mathematics. Joe particularly mentioned the use of the local environment and the use of concrete materials.

I like to take them outside and use the environment on occasion. I did it before it became fashionable with the new curriculum. For example, in relation to area, ares and hectares, I tried out surveying, showing them how to calculate ares and hectares. I took them to the GAA field and showed them that this would be equal to a hectare. That was interesting; there is nothing like showing them in reality exactly what we are talking about, for example putting gout cones in area to show exact measurement. We must always come back to the concept, to the practical side of maths being important, from infants right up to sixth class and even beyond (Appendix E.1).

4.4.3 Joe's constructivist approach to mathematical problem-solving

Initially, Joe was excited about the project and was eager to see how it might work. From the outset he felt that students with learning difficulties might become dominated by students with significant mathematical ability. According to Joe: 'The students that it is most suited to and that best grasped the topics are the good students who like a challenge and who grasp maths concepts very easily' (Appendix E.1). He continued: 'You need to be well able to read and decipher a situation' (Appendix E.1). On completion of the project, Joe decided that grouping students of similar ability might be a more appropriate way of approaching mathematical problem solving from a constructivist perspective. He said 'Children have to feel that they have something to contribute and be able to follow reasoning that is going on' (Appendix E.1).

Joe reflected on the productivity of the individual groups within his classroom and explained that there were a number of groups who were particularly productive. He went on: to explain: 'The presence of a pupil who has a flair for the subject can be a big help. He can bring the rest of the students with him' (Appendix E.1).

Joe describes constructivist teaching as 'hands on activities, using the environment and ensuring a concept does not remain abstract' (Appendix E.1). Joe revealed that it made his students more comfortable, that it provided a medium in which they could express themselves, and that they felt part of the process of coming to a solution to a problem.

A lot of the time they probably feel a bit isolated, for example, if they are just presented with a problem on their own without any idea of even when to start. If there is a group they might spark off each other, which might be helpful to each other. Some children are well able to learn like that (Appendix E.1).

Joe explained that a student is at an advantage if he/she can come to an understanding of a concept under the guidance of a teacher rather than through direct instruction. He says: 'Spoon feeding them a concept or a skill really spoils a learning opportunity. They must be left to tease it out themselves, go as far as they possibly can, and then provide guidance where it is required' (Appendix E.1). This has its advantages in that 'it makes them realise that if you read the problem first eventually it might be possible to find a starting point without having someone having to be present all of the time' (Appendix E.1).

4.4.3.1 Joe's illustration of students' experiences of learning from a constructivist perspective

Joe revealed that students enjoyed mathematical problem-solving from a constructivist perspective. He said: 'There was an element of fun in them, they were challenged' (Appendix E.1). Importantly, he noted that 'they got a certain amount of satisfaction out of being able to solve a number of these problems that they originally thought they would not be able for' (Appendix E.1). Joe emphasised that students enjoyed problems that took them out of the classroom and into the environment to solve: 'They liked those problems that they gather the information themselves before they went to solve the problems. Any kind of equipment brings both enjoyment and challenge' (Appendix E.1).

Joe admitted that it took some time to become accustomed to the format of the lessons, but any concerns he may have had were alleviated, as he was assured children were 'learning so it is very valuable' (Appendix E.1). He added: 'Outside of concepts and skills there are a lot of techniques they are developing within themselves and communicating with their friends which is important too' (Appendix E.1). Joe felt that the amount of material within the current curriculum, together with the need to cope with the varying learning styles in

the classroom, makes the delivery of mathematical problem-solving lessons from a constructivist perspective challenging. In addition, he said: 'A lot would depend on the topic too, and what kind of a problem you can create for them that would accommodate most of the group' (Appendix E.1).

Joe explained the difficulties, from his perspective, of approaching mathematical problem-solving from a constructivist perspective on a very regular basis.

> One has to be aware of the children who are average and who need one-on-one time. They need a certain amount of direct teaching to bring them on. The other challenge is good but we have to be aware that some will get lost and not all children will have the same amount of input. It can be easy for some to sit back and think that this is a bit of free time. There will always be one or two who will carry the can for the others. Making them all participate would be difficult all of the time. That's why we need the more traditional teacher directed activity as well (Appendix E.1).

Joe added that class size is also a significant issue: 'Firstly you would know the pupil very well indeed and, secondly, you would get a lot more done. You would be able to monitor everyone's work very frequently; in a class of 35 it is difficult' (Appendix E.1).

In his final comment Joe added: 'I never saw maths explored in that way before, I was very impressed. It changes normal teaching and I like giving more opportunity or ownership to students. I saw that it was a great boost to students to do a project or solve a problem completely on their own with guidance rather than direct teaching' (Appendix E.1).

4.4.4 Joe's students' perspectives of their mathematics education

Children were encouraged to talk openly and honestly about their beliefs and experiences regarding mathematics education and the primary school. From their comments, children do not see the subject as one of their favourites. They did, however acknowledge that it is necessary to study the subject. One student

declared: 'You need maths in everything so you might as well learn it while you can because then you might never understand anything' (Appendix E.2). Children revealed that they regularly feel unchallenged and one student explained: 'It makes me feel bored' (Appendix E.2). Students reported that they feel a significant amount of confusion at times, especially when they are solving problems. In further discussion, one student revealed: 'I like maths sometimes but it gets very confusing. I like it because it puts my mind to work' (Appendix E.2). One student claimed to 'hate' (Appendix E.2) the subject, going on to comment: 'When we get lots from the book and I can't do it, I really hate it' (Appendix E.2).

Students were asked to reveal areas of mathematics that they like to study, or to describe some mathematics lessons that they had really enjoyed in the past. Students indicated their considerable enjoyment when working with concrete materials. Students mentioned metre sticks and the trundle wheel. Another student indicated his interest in data and, in particular, the construction of bar graphs. Students agreed in discussion a shared interest in playing mathematics games and having mathematics contests. They mentioned when a teacher challenged them to 'beat the clock' on a number of occasions. They indicated their enjoyment of this. Students also revealed their interest in working with partners or in group situations for mathematics lessons.

4.4.4.1 Joe's students' perspectives of their mathematics lessons

The following is a description, from the students' perspective, of a typical mathematics lesson in their class. Students revealed the following:

- 'We correct our homework, teacher writes things on the board and we do them.' (Appendix E.2)
- 'We take out a book, if we are starting something new he will do a sum and then we go off by ourselves and do it. I'm often stuck.' (Appendix E.2)

Students explained that they work on their own for the vast majority of time in the mathematics classroom, and that this has been the case for their time in the

senior school. One student commented: 'We haven't worked in groups or pairs this year' (Appendix E.2). When questioned about the way they learn mathematics, one student revealed: 'It's hard because you can't talk about what you don't know, so then you get stuck very easily' (Appendix E.2). Students explained that the majority of their work comes from textbooks and that their teacher uses a variety of textbooks over the course of the school term.

4.4.4.2 Joe's students' illustrations of mathematical problem-solving from a constructivist perspective

Students' initial response was to explain that 'the problems are hard but it's easier when you are in groups' (Appendix E.2). One student continued: 'You can talk about it and other people also have ideas that might give you an idea as well' (Appendix E.2). Another explained: 'Its a lot easier because you have nowhere to go when you are stuck on your own, but you can ask for help with a group or decide on what to do with other people, and that helps you out too' (Appendix E.2).

Students were asked to describe their interactions during the mathematical problem- solving sessions they engaged in from a constructivist perspective. Initially, one student explained: 'We found it hard to come up with plans because we usually just do the work' (Appendix E.2). Students explained that some of the problems were long and difficult to interpret but that this did not pose significant difficulty because group discussion allowed students to decipher difficult problems. Another student continued to describe these situations as 'challenging and interesting when you are in a group situation' (Appendix E.2).

In conversation concerning problems, the type of which they might not have come in contact with previously, a student explained: 'We read it over and over and kind of talked about it. We just tried different things until we thought we had an answer' (Appendix E.2). Students claimed this problem was relatively easy to overcome as a member of the group was 'bound to have an idea' (Appendix E.2). Students declared they enjoyed this approach to problem

solving: 'We weren't afraid of being yelled at because people had different ways of doing the questions' (Appendix E.2). Students continued to reveal that the teacher had encouraged them to try as many different and varied methods as they wished. Students approached the problems by initially reading them, talking about them together, and, interestingly, coming up with individual plans of solution before combining all ideas to come up with a general plan of solution.

4.4.4.3 Joe's students' reflections

On reflection, students discussed the level of interest in the lesson amongst themselves and their classmates. One student described the lessons as 'a lot more fun' (Appendix E.2). Students felt less pressure when asked to do a limited amount of problems and spend the time coming up with a solution to the problem rather than focussing on getting a particular amount of problems completed. Students revealed that mathematical problem-solving 'is not all about doing sums on paper' (Appendix E.2). They also declared: 'There might be different ways to do one problem and that is ok if the answer seems right' (Appendix E.2).

4.4.5 Joe's mathematical lessons from a constructivist perspective

The following are mathematical problem-solving lessons conducted from Joe's perception of a constructivist problem-solving lesson.

4.4.5.1 Problem 1

Mark (from Sydney, Australia) and Hans (from Berlin, Germany) often communicate with each other using 'chat' on the Internet. They have to log on to the Internet at the same time to be able to chat. To find a suitable time to chat, Mark looked up a chart of world times and found the following:

At 7 p. m. in Sydney, what time is it in Berlin?

Greenwich 12 midnight

Berlin 1 a. m.

Sydney 10 a. m.

Mark and Hans are not able to chat between 9 a. m. and 4:30 p. m. their local times, as they have to go to school. Also, from 11 p. m. till 7 a. m., their local times, they won't be able to chat because they will be sleeping. When would be a good time for Mark and Hans to chat?

Teacher: After you read the problem just jot down quickly what information you might need to solve the problem, problems that you have done that were similar to these, and anything that comes into your head about the problem.

Teacher: Has anyone experience of time difference from their summer holidays?

Student A: Yes, when I go it's usually an hour ahead of Ireland.

Students spend time silently figuring out the problem on their own initially (8 minutes).

Teacher: My first idea would be to draw two clocks and colour in the times they would not be able to talk to one another, and look at and compare the clocks to figure out when might be a good time to talk.

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Students proceed to do this silently.

Teacher: Now, when Mark comes home from school at 4.30 in the evening what time is it for Hans?

Student A: Nine hours difference

Student B: He couldn't talk, no one would answer it would be 1.30 in the morning.

Teacher: Is Berlin before or after Sydney?

Student A: After, no before

Teacher: So, when Mark gets home from school, what is Hans doing?

Student B: It would be 1.30 so he would be asleep.

Teacher: But before means going back.

Student C: So it is 7 o'clock.

Teacher: Would he be able to chat at 7.30.

Student B: Yes

Teacher: What about 5.30 p.m. Australian time?

Student A: That would be 8.30 a. m. Berlin, a good time.

Teacher: So between what hours would be good for them?

Quiet discussion

Student A: From 7.30 to 8.30 in Berlin and from 4.30 to 5.30 in Australia

Teacher: Why did ye find that difficult?

Student A: We had never done a problem like it before. We mixed up going ahead in times rather than going backwards.

Joe began this problem-solving session by asking students to reflect on the given situation and to recall similar problems that they may have solved in the past. In an effort to help students realise the differences in times around the world, Joe asked the children to recall any holidays they had experienced and the time differences that existed in those places. This was the students' first experience of problem-solving in a group situation and they did not cooperate as a group during the initial phases of the lesson. Students spent eight minutes in the group situation silently working alone on the problem. The teacher offered

his opinion on the solution to the problem during this period and the children proceeded to follow his example. Students were unable to devise a problem-solving approach to this problem collaboratively, and the teacher did not encourage students to discuss any ideas they might have had to facilitate their development of these ideas.

Joe proceeded to ask lower order questions of pupils such as:

- When Mark comes home from school at 4.30 in the evening, what time is it for Hans?
- Is Berlin before or after Sydney?
- Would he be able to chat at 7.30?

In answering these questions students arrived at an answer to the problem as illustrated above. Students were not afforded opportunities to solve the problem according to Polya's (1945) four-stage problem solving procedure.

4.4.5.2 Problem 2

A grocery store has a sale on bananas. If you buy 6 bananas you get the sale price. If a grocer has 489 bananas how many can he sell at his sale price? In this case how many can be sold at the regular price?

Student A: What are the prices? We don't know.

Teacher: Ok, where do you start? What is the important information now? What numbers do you need to focus on?

Student B: Ok, 489 bananas altogether, divided by 6.

Student A: That is 81 remainder 3.

Student B: Ok, so now, what is the question?

Student C: How many bunches of 6 bananas can he sell?

Student B: We have all got 81 remainder 3 but is that it? This is confusing.

Student C: Teacher, this is all we have so is it 81 now, we don't know?

Teacher: So, what did you do? You divided 489 by 6 and got 81 remainder 3. Now put that in a sentence for me.

Student B: I know, 81 bunches and 3 left over.

Student A and C: We still don't know if that is right.

Student B: It is 81 6's and 3 left over.

Student A: He can have 81 bunches of bananas and have 3 left over for the regular price.

Joe began this problem-solving session by requesting students to consider the important information contained in the problem: 'Ok, where do you start, what is the important information now? What numbers do you need to focus on?' Students A, B and C proceeded to engage with each other without significant teacher interaction in their solving of the problem. When students requested information from the teacher, rather than supply them with this information Joe asked them to put their mathematical answer '81 remainder 3' into a sentence. Students then proceeded to state: 'He can have 81 bunches of bananas and have 3 left over for the regular price'.

4.4.5.3 Problem 3

Divide the face of the clock into three parts with two lines so that the sum of the numbers in the three parts are equal.

Student A, B and C: Do we draw the clock?

Student B: Yes

Student A: Now will we put the numbers?

Student C: I think we just need the 12, 3, 6 and 9.

Student A: Ok now, we do the lines, we can just rub them out if we need to.

Student B: I think we need all the numbers.

Student A, B and C: Yeah, let's put all of them in.

Student A: We can only use 2 lines to make 3 parts remember.

Student B: Oh, only two lines, like the hands of a clock is it?

Student A, B and C: So 2 lines making 3 parts

Student C: We should all try making one and then see which one works.

Student B: What about from 12 to 6 and from 3 to 9?

Student C: No, because look then it is in 4 parts.

Student C: Oh I see, we have to have all of the numbers so we can add them up.

Teacher: Yes

Student A: 87 is the numbers all added together.

Student B: Put the 9 with the 8 and the 7.

Student A: That is 24.

Student B: Where does the 12 go?

Student C: With the 10 and 11

Student B: It might make more sense for it to go with the 1 2 3 4 5, the smaller numbers.

Student B: Put the 9 and the 8 together with the 7.

Student A: That means that is 27.

Students send time working silently on task.

Student C: I think it's 12 with all of them 1 2 3 4 5, then the 6 7 8 9 and the 11 and 12.

Student A: All the 3 parts are not the same.

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Student B: I think I have it, add up all the numbers on the clock together.

Student C: The answer is 78 divided by 3 is 26.

Student B: Wait! So we need to put the clock into 26 parts.

Student A: No each part has to be 26.

Teacher: 'Student X' is going to explain to the group.

Student A: What, but we nearly have it.

Teacher: Ok, but just listen to 'Student X'.

Teacher: 'Student X' has made sure that 10 9 3 and 4 are in one part of the clock and they add up to 26. She made sure that 11 12 1 and 2 are in another part, they also add up to 26 and then 8 7 6 and 5 are left and they make 26.

Student A, B and C: Ok so.

Joe's students' work is displayed below.

Figure 12: Joe's students' work (Problem 3)
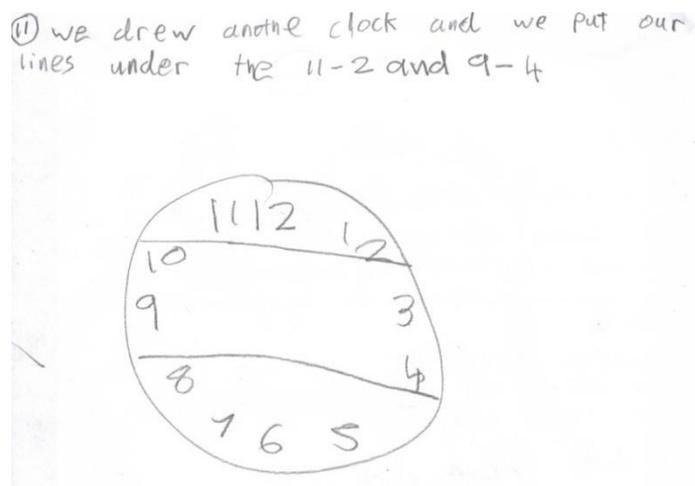
Students A, B and C began solving this problem without interacting with the teacher. Students discussed their solution strategies openly with one another from the outset. The students' understanding of the problem was unclear at the outset, and this is evidenced from the initial conversation.

Student B: Only two lines, like the hands of a clock is it?

Student B: What about from 12 to 6 and from 3 to 9?

Student C: No,because look then it is in 4 parts.

As students proceeded, they discovered an effective strategy for solving the problem.

Student B: I think I have it, add up all the numbers on the clock together.

Student C: The answer is 78 divided by 3 is 26.

Student B: Wait! So we need to put the clock into 26 parts.

Student A: No each part has to be 26.

At this stage the teacher intervenes and asks Student X, who has designed an effective strategy for solving the problem, to explain his strategy to the rest of the class. This stops the students who were engaged in developing an effective problem-solving strategy from completing their work. Joe concludes by repeating the solution offered by Student X.

4.5 Participant four: Tomás

Tomás was a participating teacher in a Limerick city school. The following is a photograph of Tomás' classroom.

Photograph 4: Tomás' classroom

4.5.1 Tomás' profile

Tomás is a fourth class primary teacher and has been in this position for two years. He has taught at primary level for six years having experience in first and second classes. Tomás has an undergraduate degree in media and communications and a postgraduate diploma in primary education. Tomás chose to return to teaching because, he said: 'I always had an interest in education stemming from a heavy family background in it' (Appendix F.1). Tomás showed a significant interest in the project from the outset and it was obvious that constructivism already played a significant role in his teaching of mathematics. During interview, Tomás outlined his strategies and approaches for teaching mathematics and they are clearly constructivist.

Tomás has a real interest in the subject and this stems from his experiences at second level. He explained: 'I enjoyed it more at second level than at primary level. The teachers teaching it were specialised in maths and that is the reason I think, their methods were better, more refined' (Appendix F.1). Tomás' recollection of his primary years was vague yet he recalled positive mathematical experiences at fourth class level: 'It was a mixed bag of experiences both positive and negative' (Appendix F.1). He explained that problem-solving was a significant part of their mathematics lessons at fourth class level: 'Her lessons were very much similar to what we are encouraged to do now in terms of using resources and having children solve problems' (Appendix F.1). He continued: 'Teachers never really went beyond the pages of the textbook, my fourth class teacher did, that's why she stands out' (Appendix F.1).

4.5.2 Tomás' teaching of mathematics

Tomás described his experiences of learning to teach mathematics at third level as basic: 'There was very little opportunity to go into anything in great depth. We moved from strand to strand too quickly' (Appendix F.1). Tomás returned to complete his graduate diploma in education as a mature student and noted: 'A lot of adults, when they return to education at 21 or 22 or even 42 have forgotten an awful lot of what they did at primary school. A lot of older students were unsure of the concepts themselves. They needed to brush up on that' (Appendix F.1). Tomás went on to recommend: 'I think a two fold approach is the way forward, similar to the way Irish is approached in the education colleges. In one sense you should be taught to teach it and, in another, you should be taught the subject itself' (Appendix F.1). In conversation, Tomás detailed that there was little opportunity afforded to students to study mathematics education at third level in any great detail. Again, Tomás mentioned that teachers at second level may be more equipped to teach the subject as they have a more significant mathematical background.

Initially, Tomás was asked to describe, in his opinion, the core principles behind the mathematics curriculum. He said: 'It is child centred. Whatever their

experience or expertise, they do have the opportunity to partake fully in the lesson. Every child can have a part in every lesson because of the methodologies that are suggested' (Appendix F.1).

Tomás believes a child's understanding of the number facts is 'crucial' (Appendix F.1) to his/her mathematical development. He said: 'I drill them from the weakest to the strongest child. On a daily basis we play table games so that they are well accustomed to their number facts. You can know your methods inside out but if your tables let you down you are at an extreme disadvantage' (Appendix F.1). Problem-solving plays a significant role in his classroom because, he explained: 'Society needs people who can work around a problem and see things in a number of different ways' (Appendix F.1). Tomás acknowledges the emphasis on problem-solving in the primary curriculum. In the past, he explained: 'Problem-solving skills were not emphasised when they should have been because of overload' (Appendix F.1).

Tomás enjoys teaching mathematics. He employs a variety of strategies and methodologies in his exploration of the subject. Tomás' explanation of his approach to mathematics lessons is constructivist. He provides opportunities for children to approach a new concept from their own particular level of understanding.

> One of the fundamentals is that when we are starting a new topic, I point out that even when they end up with the right answer, if they haven't used the right method then clearly they are still right in the method they have used. We then discuss what different methods you could use to come up with an answer. In other words, we go through all the maths language that you might have in the different concepts. Before I narrow it down to the concept recommended by school policy, renaming for example, I emphasise that there is often more than one route. The children enjoy that in that they use their own experiences to help them understand (Appendix F.1).

Tomás went on to explain that he places more significance on the method utilised to achieve an answer than on the answer itself. He explained: 'Once they are in range of achieving an answer, I get them to explore the method by

which they arrived there' (Appendix F.1). Consequently, Tomás explained, discussion plays an important role in the mathematics class: 'I get them to go through it themselves and talk about it themselves' (Appendix F.1). Tomás further develops a pupil's understanding of a topic by encouraging them to teach one another concepts that have already been explored by the teacher. He said: 'Sometimes if we are coming back to a concept, I get one child to come up and actually teach it' (Appendix F.1). Pupils work co-operatively in groups for problem-solving and when working with concrete materials in Tomás' classroom.

When asked to explain, in his opinion, what a good mathematics student can do, Tomás revealed that their capacity for problem-solving was an extremely significant indicator. He explained that a weak student can have a 'decent enough' (Appendix F.1) capability in relation to computation because of the emphasis on mathematical number facts in the primary school' (Appendix F.1) but the able student is one who can cope with 'unusual mathematical problems' (Appendix F.1). He explained that problem solving: '… delineates the good mathematics student from the particularly strong mathematics student' (Appendix F.1). 'He is also methodical, picks up on new concepts and questions' (Appendix F.1).

4.5.3 Tomás' constructivist approach to mathematical problem solving

Tomás has a clear understanding of why constructivism is central to the Primary Mathematics Curriculum (1999). He said: 'To become independent, children must collaborate and use their knowledge in problem situations' (Appendix F.1). Tomás went on to describe constructivism as taking 'the child's own experiences and, through interaction with others, you broaden and develop their knowledge base so they become more accustomed to more complex and different problems and methods' (Appendix F.1).

Tomás found that the mathematically able student was very challenged and interested in approaching mathematical problems in collaboration with their peers. Tomás felt that being a recently qualified teacher helped in the

management and co-ordination of these lessons because he was familiar with setting successful group work tasks and activities from his time as a student teacher.

Tomás approached his mathematical problem-solving lessons in the following manner. Students were divided into groups of mixed abilities and presented with a mathematical problem displayed on an overhead projector. Students were encouraged to follow Polya's (1945) four-stage problem solving procedure. Tomás drew students' particular attention to the need for examining the problem for relevant and irrelevant information. Tomás explained: 'After ten minutes of a session, I ask children to explain strategies they may have chosen to solve problems to give students who may be having difficulty, ideas about maybe where to begin' (Appendix F.1).

The challenge, Tomás revealed, is to keep students with lower levels of mathematical ability challenged during problem-solving sessions. Tomás achieved this, in his opinion, by giving an increased amount of guidance in groups of similar levels of ability.

In mathematics, you will have people who will work at a very fast pace, way ahead of others in their group so another way might be to put students of a similar ability together. That works quite well as they can bounce off each other more. Obviously the lower ability students need some guidance and I give them that. I use the same questions but I work more with the weaker students (Appendix F.1).

Tomás explained that he had to 'actually teach problem solving' (Appendix F.1) and 'go back and do simple examples with them' (Appendix F.1). Tomás continued: 'It's not the problems the teacher needs to be conscious of, rather the methodology or the teaching of it' (Appendix F.1). Tomás revealed that teachers need to become aware of developing pupils' problem-solving abilities: 'We spend too much time on computation, which is somewhat useful but really, of how much use is it compared with the ability to problem solve' (Appendix F.1).

4.5.3.1 Tomás' illustration of student experiences of learning from a constructivist perspective

Tomás explained that he witnessed a 'significant improvement in students' problem-solving skills' (Appendix F.1) and would 'highly recommend approaching problem-solving in this manner' (Appendix F.1).

> It opened their eyes to problem-solving. They have become more analytical in their thinking. They are able to apply strategies they have learned outside of the typical mathematics class. I suppose also, it has sparked a general interest in problem solving. It has become an integral part of my approach to the teaching of mathematics (Appendix F.1).

Tomás revealed that his students 'were very taken by it' (Appendix F.1) when asked about their initial reactions to the project. In discussion, it became clear that students whom Tomás described as being at the 'top end' (Appendix F.1) of the class had no difficulty in interacting with each other in group situations to solve problems. Tomás described their interactions as 'high intensity'. Students were eager to solve problems so Tomás decided short lessons of thirty minutes in duration were more appropriate from a management perspective. He said: 'It is high intensity stuff so to keep them motivated, interesting lessons that don't drag on are much more appropriate. They need to be kept motivated and busy and that is difficult work' (Appendix F.1).

Tomás found that he had to give guidance in particular to students with lower levels of ability. Tomás found he had to go into detail with the explanation of problem-solving procedures with these students. Tomás highlighted procedures such as searching for relevant and irrelevant information and re-examining the problem for important information.

4.5.4 Tomás' students' perspectives of their mathematics education

Students were initially asked what they felt about the subject mathematics and their experiences of mathematics on a daily basis at school. The following were some of the replies:

- 'I much prefer to do subjects like Physical Education. It gets hard and frustrating when you forget about method and things like that.' (Appendix F.2)

- 'Sometimes it can be all right when he puts it in a fun way rather than a boring way like going outside, like when we used the wheel outside to measure centimetres.' (Appendix F.2).

- 'I find it really hard trying to get it and I can't get it, I can multiply though.' (Appendix F.2).

- 'I used to enjoy it when I was smaller but now, not so much.' (Appendix F.2).

- 'I like multiplication because we have a game for doing i.' (Appendix F.2).

- 'Sometimes he (Tomás) brings in a whole new method all of a sudden and that is hard.' (Appendix F.2).

When asked to describe any particular aspect of mathematics that they enjoyed, two students revealed: 'I like when we work together' (Appendix F.2). Other students listed enjoying studying the strand unit data, playing mathematical games, and problem solving in co-operation with their peers. Students explained that when working with data, they carry out surveys and make charts based on the results. Students were also asked to outline what particular aspects of mathematics they disliked. They indicated that 'tests can be very tiring and there is lots of pressure' (Appendix F.2). One student added: 'And lots of multiplication and division sums' (Appendix F.2). Another student explained: 'We keep doing it (mathematics) for a long time' (Appendix F.2).

4.5.4.1 Tomás' students' perspectives of their mathematics lessons

Students described the teacher's teaching of mathematics from their perspective. One said: 'We do most of our work individually and its multiplication and division and things like that' (Appendix F.2). Another said: 'We do problem solving and things like that in pairs' (Appendix F.2). Students agreed: 'Normally he (Tomás) tells us what sums to do in our books and we do them on our own' (Appendix F.2). 'We usually do lots of sums that take only about a minute'.

4.5.4.2 Tomás' students' illustrations of mathematical problem-solving from a constructivist perspective

Students' initial responses to questions related to exploring mathematical problem-solving from a constructivist perspective were extremely positive. One student explained: 'The one thing I did like was that it improved our thinking about different things, we thought maths weren't in things like dressing dolls' (Appendix F.2). Other responses were:

- 'It was nice because you could work with your friends and if you couldn't get it you could ask one of them.' (Appendix F.2)
- 'You are comfortable with your friends.' (Appendix F.2)
- 'It's better because it actually gives you a challenge.' (Appendix F.2)
- 'You have to break things down and kind of investigate.' (Appendix F.2)
- 'The problems were nice because they weren't like the ones that are in the books. I'd like to see more of them in the books.' (Appendix F.2)
- 'There wasn't so much pressure on you so it was good.' (Appendix F.2).

Students explained that devising strategies to solve the problems was achievable when they discussed the problems. One group explained: 'It was easy most of the time to come up with ideas ourselves, I think we only had to ask the teacher once about one thing' (Appendix F.2). Other students revealed that there was opportunity for individual work within the group situation but that 'we could

ask for help when we wanted it' (Appendix F.2). One student explained: 'It's harder individually but in a group you can read the question and keep asking each other things that are important' (Appendix F.2). Interestingly, one student said: 'Sometimes the teacher might be doing something for a week and you still might not know it but with your friends, they can explain it easier' (Appendix F.2).

In their approach to solving the problem students explained that they read the question and 'most of them were similar to what we had before and we could talk about those ones and remember what we did' (Appendix F.2). Students revealed that they concentrated on what was required in the situations and that 'not everything is needed' (Appendix F.2). Students explained that there was no timeframe in which they had to be finished, and that they were only asked to do a limited number of problems and not a vast amount of computation: 'There wasn't so much pressure on you so it was good'.

One student explained that he experienced frustration during the initial lessons but that after a period of time, and because they were only required to spend time at one particular problem per lesson, 'with others help it was easier' (Appendix F.2). Another continued: 'It was hard at first but now we know what we are doing and we don't talk at the same time, everyone gets a chance to talk' (Appendix F.2). After the initial lessons students revealed they realised they must 'talk about the important stuff and try different things if some didn't work' (Appendix F.2). One student likened the development of his problem-solving abilities to playing a video game, 'when you go up through the levels, they get harder but you also get better at the game' (Appendix F.2). He continued: 'I learned a lot, they seemed easy but they were hard because of the detail, they make your mind work' (Appendix F.2). Another added: 'It's interesting when there is story involved' (Appendix F.2). In concluding his interview, one student highlighted that 'you might be able to do different things that you never saw before' (Appendix F.2).

4.5.4.3 Tomás' students' reflections

Students were asked to offer advice to those who might be unfamiliar with exploring mathematical problem-solving in the manner they had become accustomed to. The following is a selection of their comments:

- 'Take your time and be careful what you do.' (Appendix F.2)
- 'You don't need all of the information all of the time.' (Appendix F.2)
- 'Try working it out even if it might be hard.' (Appendix F.2)
- 'It's ok to get it wrong and ask for help.' (Appendix F.2)
- 'Think about things first before you do things; read it a few times.' (Appendix F.2).

4.5.5 Tomás' mathematical lessons from a constructivist perspective

The following are mathematical problem-solving lessons conducted from Tomás' perception of a constructivist problem-solving lesson.

4.5.5.1 Problem 1

A farmer looks out into his barnyard and counts 14 heads – some horses and some chickens. He also counts a total of 40 legs among his animals. Can you figure out how many horses and how many chickens must have been in the barnyard?

Teacher: Remember all to pick out the important information and not to rush. Take your time and don't be afraid to try anything. Off you go in your groups now.

Student A: Well there are definitely 14 animals anyway because it says 14 heads.

Student B: Yes, and chickens and horses only have 1 head.

Student C: 32 feet, so a horse has 4 and a chicken has 2.

Student B: Pick a random number so 7 horses and 7 chickens and that's it.

Student A: No that wouldn't figure out it is 28 and 14 which is too much, so less horses.

Student B: 6 horses and 8 chickens? 6 horses is 24 and 8 chickens is 16. No that is still too much.

Student A: Maybe 5 horses and 9 chickens

Student B: No

Student C: Is it am, 3 horses which is 12 and 11 so plus 22 which is 34?

Student B: 5 and 9 no, that wouldn't work either.

Student A: What is 2 horses, it is 8 and then

Student B: 12 chickens which is 24

Student C: That's it, 32 we got it.

Student A, B and C: It is 2 horses and 12 chickens.

Student A: Teacher, if 2 horses is 8 legs and 12 chickens is 24 so 32.

Teacher: Well done, how did you do it?

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Student B: We just guessed it and found that the heads added to 14 and the legs to 32.

Tomás began this problem-solving lesson by ensuring children understood the procedures for mathematical problem-solving. Tomás reminded children of the importance of reflecting on the problem: 'Remember all to pick out the important information and not to rush. Take your time and don't be afraid to try anything'. Students chose to use trial and error in their efforts to find a solution to the problem. Students were convinced during the initial stages of the lessons that there were 14 animals in the barnyard as they equated, correctly, 14 heads with 14 animals. Students were quick to realise the difference lay in the amount of feet. Students in the group swiftly decided to put various quantities of animals together in an effort to equate 14 heads and 40 animals. Students engaged in rich discussion with every student having an opinion on the problem. It appears that children utilized their mental mathematics skills effectively. Students were quick to identify a solution and notify their teacher. The teacher concluded the lesson by requesting students to explain how they solved the problem.

4.5.5.2 Problem 2

A grocery store has a sale on bananas. If you buy 6 bananas you get the sale price. If a grocer has 489 bananas how many can he sell at his sale price?

Teacher: Ok, where do you start, what is the important information now? What numbers do you need to focus on? What information is important and what information is unimportant? Use highlighters.

Student B: Ok, 489 bananas altogether, divided by 6.

Student A: Will we underline the important information so we will be able to go back to the important information when we need to?

Student C: So underline 6 and 489.

Student B: Now divide 489 by 6.

Student A and C: Why?

Student B: Because we need to find out how many bundles or bunches of 6 are in the 489.

Student A: I'll do it here; we will all do it to be sure. It is 81 with 3 left over.

Student C: How many bunches of 6 bananas can he sell?

Student B: We have all got 81 with 3 left over.

Student C: Teacher, this is all we have, so is it 81.

Teacher: Ok, think back to the bananas what does that 81 actually mean now and then we will look at the 3.

Student B: It means 81 bunches or 81 6's.

Student A: It is 81 6's and 3 left over.

Student A: He can have 81 bunches of bananas and have 3 left over for the regular price.

Student B and C: That's it.

Teacher: Yes, very good.

Again the teacher focussed student attention on the need for reflection during the initial stages of the problem-solving lesson. The teacher reminded students to search for important information, for unimportant information, and to use their highlighters if necessary. Student B appeared to dominate during this particular lesson as he identified quite quickly that by performing the division operation that the problem could be solved. Students A and C had difficulty in understanding this but student B revealed clearly why this was necessary for Students A and C. Having achieved an answer of '81 remainder 3' students had difficulty in interpreting that answer. The teacher posed an interesting question for the group giving them the stimulus to come to a correct answer. The teacher asked them to make sense of the answer 'what does that 81 actually mean'. Students A, B and C were then quite capable of interpreting the mathematical answer for the teacher with student A revealing, 'he can have 81 bunches of bananas and have 3 left over for the regular price'. This question the teacher posed was instrumental in students arriving at this conclusion.

4.5.5.3 Problem 3

A man has to be at work by 9 a. m. and it takes him 15 minutes to get dressed, 20 minutes to eat and 35 minutes to walk to work. What time should he get up?

Teacher: This is a short problem; but still, remember to check for information that may be important and unimportant and talk about anything that comes into your head about the problem with your group.

Student A: If he leaves for work at twenty-five past eight it will take him 35 minutes to get to work on the dot.

Student B: No! Start again because the rest didn't hear.

Student C: We have to add 15 minutes, 20 minutes and 35 minutes together OK?

Teacher: Ok, quiet now and listen to his idea

Student C: That is 70 is it?

Student A: Yeah

Student B: So what time is 70 minutes before that?

Student C: Well before 8 because 70 is more than an hour.

Student A, B and C: Yeah ten to eight so

Student B: He leaves at twenty-five past eight. He gets dressed. No! 20 minutes to eat so five past eight and then 15 before that is ten to eight.

Teacher: Student A, would you explain to us how you got your answer? Listen to the question everyone again.

Proceeds to read the question.

Off you go Student A

Student A:You add 20, 15 and 35 and you get 70 and 70 is one hr and 10 minutes and take that away from nine o'clock is 7.50.

Teacher: That is ten to eight. 60 minutes is one hour so, therefore, that is 8 o'clock and then 10 minutes before 8 is 7.50 as Student A said. Very good.

Similar to the directions given to students on other occasions, the students were asked to 'check for information that may be important and unimportant and talk about anything that comes into your head about the problem with your group'.

Students appear to work particularly well as a group because one student explained: 'No! Start again because the rest didn't hear'. The teacher on this occasion interrupted the group's activities more frequently just to focus their attention on the problem at hand. Students progressed well and questioned each other on their ideas. Students worked methodically and utilized subtraction to solve the problem. The teacher concluded the lesson by asking one member of the group to explain their answer and repeating the solution to the problem the pupils achieved.

4.6 Participant five: Mike

Mike was a participating teacher in a County Kildare school. The following is a photograph of Mike's classroom

Photograph 5: Mike's classroom

4.6.1 Mike's profile

Mike is a primary teacher with eight years classroom experience ranging from senior infants to sixth class levels. Mike also spent one year as a resource teacher for non-English speaking pupils. Mike has both undergraduate and post-graduate qualifications. He has an undergraduate degree in primary education and a Master of Arts degree in Language Education (German). From the commencement of the project, Mike displayed great enthusiasm and interest. Mike has a keen awareness of issues and trends relating to primary education, as he is a staff representative on the school Board of Management and is also local Branch Secretary of the Irish National Teachers Organisation for his district.

Mike was very open and forthright about the project as it progressed; this is also very evident in his final interview.

Mike admitted that mathematics was not one of his favourite subjects at primary level. He attributes this to a view in his own ability: 'On balance, I think I would have been a very average student at primary school. I didn't struggle at maths but I wasn't brilliant, I was average' (Appendix G.1). Mike recalled his days in primary level. He said: 'We sat in rows and worked at sums that were on the board and sheets of them. It was constant repetition of the same sort of sum, addition, subtraction, multiplication and division' (Appendix G.1). He was taught in group situations according to ability. The transition for primary level to second level was smooth and he chose to study mathematics at honours level until the completion of his Junior Certificate, and then at pass level to Leaving Certificate Level. Mike highlighted a positive correlation between a student's attitude towards a subject and a teacher's style of teaching the subject: 'If you like their style and their teaching of maths, you will do well' (Appendix G.1).

Mike recalled his experiences of learning to teach primary mathematics at third level. Mike found he had to do extensive research on topics that he was teaching. He recalled: 'We were given handouts and schemes and you were told to go teach them on teaching practice' (Appendix G.1). Mike feels that a more concrete, hands-on approach would be of greater benefit to students.

4.6.2 Mike's teaching of mathematics

Mike describes the mathematics curriculum as 'hands-on' (Appendix G.1). He feels it allows teachers to get children 'active' (Appendix G.1) and 'engaged' (Appendix G.1) in their learning. Mike likes the mathematics curriculum especially, 'anything that can be integrated with another subject area and makes maths alive' (Appendix G.1).

Mike allocates at least one hour per day to mathematics, as 'it moves quite slowly because of the range of abilities in the classroom' (Appendix G.1). In discussion around dealing with the range of ability levels in his classroom, Mike

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explained that he tries to give special attention to the students with specific learning needs, but that a large student population in his classroom restricts him in this. It has also implications, in his view, for keeping exceptionally able students challenged. When asked about fostering the development of high achieving pupils, Mike explained: 'With the numbers it's quite difficult. It can be a challenge to get past the basics that need to be done. I suppose you can just provide material for them and try get them to work on their own' (Appendix G.1).

Mike is passionate about the lack of realism in requiring teachers to teach the curriculum as outlined to large groups of students.

> If you have a large class and try to differentiate the activities, it is hard to make progress. Are we to plan for all on an individual basis? People have to be practical and get real and acknowledge the problems in today's classrooms. Perhaps we could consider streaming down the line, using the resource teacher and coming together so that we can really attend to the needs of all. I've explored this with management but to no avail (Appendix G.1).

Mike constantly monitors student work, which is presented in copybooks. He highlighted the importance of mathematical language.

> It is important that the language we use is standard; yet we must vary the way questions might be asked. They might just grasp a concept but when it is presented to them in alternative format, it might just evade them. The more variants you give them in the questions the better. I suppose differentiation is key (Appendix G.1).

Mike explained that several factors, at times, overwhelmed him in relation to the teaching of mathematics: 'You're trying to cover mental maths, a programme of work that has been given to you by the school, and you're also trying to problem solve' (Appendix G.1). Mike attributes this to the lack of cohesion between school authorities and external agencies such as the Primary Professional Development Service (PPDS). Mike feels that every partner tries

to influence the exploration of the curriculum and that, in turn, teachers can have unmanageable workloads. Mike does, however, find value in new ideas and approaches modelled by agencies such as the PPDS, especially in relation to mental maths and maths games. Mike explained the necessity of covering material presented by the various textbooks: 'They need to have a basis. When they go to secondary, they will presume that a lot of material has been covered in primary … children need to have a good basis in the operations' (Appendix G.1).

Mike engages his students in mathematical problem-solving once children have a good basis in the concept. He said: 'Throughout the term and the week, as you get to Friday, you try and get them to solve the problems based on material that perhaps you would have covered' (Appendix G.1). Mike also spends significant time on mental maths which he also explains is part of his teaching of mathematics problems: 'We also do the mental maths regularly and there are a lot of written problems there that are not actually taught but they are getting practice in. They are developing their own strategies and using what they know already in those scenarios' (Appendix G.1).

Mike endeavours to integrate mathematics with other areas of the curriculum. He feels passionate about the creative arts and described a recent lesson involving angles and lines. Students' knowledge of angles and lines were incorporated into a lesson based on the construction of model Egyptian pyramids. He explained: 'The practical applications in other subject areas allow for mathematics, such as in science, making a spinner and calculating the degree of each angle' (Appendix G.1). Mike uses Information and Communications Technology (ICT) on a regular basis and also regularly discussed strategies that work well in their classrooms with other members of staff (Appendix J.1).

Mike indicated his unease with the teaching of fractions to young children. He finds it hard to justify the teaching of the multiplication of fractions and the division of fractions to young children. Unfortunately, Mike explained: 'You just have to sit down and say look, this is the rule for multiplying a decimal by a decimal and you just do it with them' (Appendix G.1).

4.6.3 Mike's constructivist approach to mathematical problem-solving

Initial conversations around constructivism were concerned with its employment on a day-to-day basis in the mathematics classroom. He said: 'I certainly see that it can play a major role in the teaching of maths. You are starting with the children themselves and their own starting point' (Appendix G.1). Mike revealed that constructivism does not play a significant part in teachers' planning for and teaching of mathematics.

> I think tradition stands out above everything at the moment. Infants yes, but bookwork takes over very quickly in first and second classes. I think bookwork takes precedence from then on. We lose sight of what's important. We need to remember that the maths book is just a resource and that it is more important to have a hands-on approach. Until we lose that mentality and of course have material prepared for such lessons, which can be time consuming, we are not really progressing. We need to take the lead from infant education. As you go up the school, constructivism becomes watered down quite a bit (Appendix G.1).

Mike describes constructivist teaching as:

> A hands on approach to a topic, your starting point is the level of understanding of the children. It is about group work, getting them to come together and work co-operatively, testing out their own ideas and theories. It has children trying out ideas and revisiting them to make changes and alterations (Appendix G.1).

Mike explained that our curriculum allows for constructivist teaching methods to be employed in the primary classroom. He describes the curriculum as a 'menu curriculum' (Appendix G.1). He feels 'pressure of textbooks' (Appendix G.1) plays a more significant part than the curriculum actually does. Mike realises that, 'it's a matter for schools to go back to the drawing boards and realise what is actually important for a child as he or she develops into adulthood in today's world' (Appendix G.1).

4.6.3.1 Mike's illustration of students' experiences of learning from a constructivist perspective

Mike was in complete agreement that it was a fully worthwhile enjoyable initiative to engage senior school pupils in constructivist learning.

> They were collaborating, testing out hypotheses, coming up with their own strategies and modifying them if necessary. Really at the end of the day, that is what we want a child to be able to do as they leave, to be able to become involved in society. Really it should be done across the school and levels as you can imagine how well they would cope with novel situations if this was how they coped now and they had never done it before (Appendix G.1).

Mike's first comment was that children found it a very enjoyable experience: 'The children were adaptable and became used to the innovation quite quickly. Group work was not a shock to them. I think they are very exposed to it at junior levels. They are not usually used to it in maths at this level' (Appendix G.1). Mike continued: 'There was a very positive vibe in the classroom' (Appendix G.1). Students were very productive in their groups, and, again, Mike mentioned children were very enthusiastic in their groups: 'They did all they were asked and enjoyed it' (Appendix G.1). In his explanation of this, Mike explained,

> I suppose it was a new approach that they had not experienced and they were quite interested in the different problems that were a deviation from the norm. I suppose everyone likes a challenge. Working together as a team will always be more stimulating for children than working on one's own. It really caught their imagination, it was new and novel (Appendix G.1).

Mike admitted that group teaching for mathematics had not played a significant part in his mathematics teaching, but that following his engagement in this project it will play a future roll. Similarly, constructivist teaching methods will be employed in Mike's classroom: 'There is great scope for and potential for the development of children. They really are encouraged to think for themselves

and rely on themselves more. I'd start them on easy problems and gradually help them towards looking at more difficult ones' (Appendix G.1). Mike also mentioned that it would be useful to introduce children to constructivist teaching methods in a gradual way, through pair work and then eventually group work.

Mike agreed that children did achieve curricular objectives related to mathematical problem-solving by collaborating, by devising their own problem solving strategies, and by solving questions. Mike revealed: 'They came up with ways of looking at problems that I didn't even think of. They were very good in that regard' (Appendix G.1).

Mike explained his nervousness in relation to 'relinquishing control' (Appendix J).

> I suppose, perhaps, I was taking a step backwards, relinquishing control and that was difficult at the start. As teachers we feel we have to be in control of the whole lesson, but we must have children test their own ideas and hypotheses. We have to obviously take a look at the role of the teacher in all this (Appendix G.1).

Mike cautioned that

> Some students with learning difficulties may have got a little lost, dominant characters always come out. The teacher must have made a sound evaluation of their understanding before setting tasks and designing groups. I think maybe ability grouping might work a little better (Appendix G.1).

Mike openly discussed his difficulties with teaching mathematical problem-solving from a constructivist perspective.

> I would have felt at times, what am I achieving here overall? At times I said to myself, I have a maths programme that does not appear to be covered in this. What is the end objective of all this, the visible results? They are not doing traditional bookwork that is expected of me by all the partners here. I think it is

ingrained in me that there must be quantifiable results visible regularly (Appendix G.1).

In his final comment, during his final interview, Mike explained why the project was beneficial for him.

> It has opened my eyes, children do like engaging with problems and with themselves. They can learn a lot from each other, from teaching each other, and by reasoning together. Next year I will engage with more problems. It has opened my eyes to presenting material, even new material, to students as they are more capable than we might actually think (Appendix G.1).

4.6.4 Mike's students' perspectives of their mathematics education

The students in Mike's classroom exhibited positive attitudes towards mathematics. When asked what their thoughts on the subject were, students replied:

- 'I think it is a good subject. You use it everyday unlike other subjects like history in a shop.' (Appendix G.2)
- 'I like it. I like chance and measurement. I like more than doing sums.' (Appendix G.2)
- 'It's interesting, it makes you think, there is always work to do. You can never be bored if the topic is interesting.' (Appendix G.2).

One student revealed: 'I think that it is a good subject because it teaches you different skills to work out different problems, not necessarily with numbers but other types of problems' (Appendix G.2). This student continued, explaining: 'You use it doing everyday things like how long you need to put on the timer for when cooking' (Appendix G.2).

When asked if there were any particular areas they disliked, one student expressed a dislike for computation. Another student said: 'They don't help you work out real problems like word problems do' (Appendix G.2). However, the

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same student commented: 'You need to be able to do the simple maths' (Appendix G.2).

When asked to describe an area of the subject that they enjoyed learning, students discussed strands such as shape and space and chance but they also appeared to enjoy mathematical problem solving because ,'sums like 28 – 12 don't really help you work out real problems like the word problems do' (Appendix G.2). Another student added: 'I like word problems that have everyday situations that I face myself sometimes, like finding better value in the shop' (Appendix G.2). Students explained that although they worked in groups during the term, 'it was like normal problem solving but just with your friends' (Appendix G.2).

4.6.4.1 Mike's student's perspectives of their mathematics lessons

Students described the teacher's teaching of mathematics from their perspective: 'We have played a lot of maths games, not for too long, and I really like them' (Appendix G.2). Students described their teacher as one who goes over topics quite well spending significant amounts of time on the number strand, highlighting in particular fractions, decimals and percentages. Students explained that they work on their own quite a lot using their mathematics textbook *Mathemagic*.

Students find that their teacher introduces them to interesting 'things' (Appendix G.2) in maths regularly, and, in particular, the topic of mathematical games came up repeatedly during interview. This correlates with Mike's own apparent interest in and employment of mathematical games in his mathematical lessons. Students revealed that they get 'enough' (Appendix G.2) homework on a very regular basis and it does not pose much difficulties for them.

In discussion, students in Mike's classroom were positive about their teacher's exploration of mathematics with them. Students enjoy working on mathematical investigations and challenges, and appear to enjoy solving mathematical problems.

4.6.4.2 Mike's students' illustrations of mathematical problem-solving from a constructivist perspective

Students' initial reactions to their participation in the initiative were extremely positive. Students declared:

- 'They were different and challenging unlike what we do.' (Appendix G.2)
- 'We talked a lot and it was interesting to work with your friends.' (Appendix G.2)
- 'The problems were fun, you could like, imagine some of them happening.' (Appendix G.2)
- 'It's easier when your friends help you out sometimes instead of the teacher.' (Appendix G.2)
- 'I like working with other people.' (Appendix G.2)
- 'It's easier to solve problems when lots of people put their ideas together than when your on your own.' (Appendix G.2).

Students explained that they solved the mathematical problems in group situations and found these experiences to be rewarding. From the data below, however, students did not appear to spend significant time on reflecting on the problem and devising a strategy of solution but rather proceeded quickly to solve the problem without reflecting or planning. From the above data it appears that students in Mike's class are eager to solve problems. However, students have not had any significant experience of reflecting on the context of the problem and devising strategies to solve it in collaboration with their peers. This eagerness, therefore, combined with their lack of experience may have made it difficult for Mike to incorporate these problem- solving stages into his teaching of mathematical problem-solving.

Students' revealed that their teacher helped them out as they solved problems. One student explained: 'The only difference was that we were in a group and not on our own' (Appendix G.2).

4.6.4.3 Mike's students' reflections

In discussion, students' revealed the following:

- 'Working together makes things faster.' (Appendix G.2)
- 'It's important to share things with your friends.' (Appendix G.2)
- 'Those kind of problems are nice and more interesting.' (Appendix G.2)
- 'There is more talk in the room.' (Appendix G.2)
- 'It prepares you better for using maths outside of school.' (Appendix G.2)
- 'I like problem solving in maths now.' (Appendix G.2)
- 'You can try different things out.' (Appendix G.2)
- 'Sometimes it can take a long time to get an answer.' (Appendix G.2).

4.6.5 Mike's mathematical lessons from a constructivist perspective

The following are mathematical problem solving-lessons conducted from Mike's perception of a constructivist problem-solving lesson.

4.6.5.1 Problem 1
Uncle Henry was driving to Cork when he spotted a big green gorilla on the side of the road. He screeched to a stop, jumped out of his car. He saw the outline of a number on the gorilla. He couldn't quite see the number, but he knew it was a 4 digit number. And:

1) He remembered seeing a number 1.

2) In the hundred's place he remembers the number is 3 times the number in the thousand's place.

3) He said the number in the ones place is 4 times the number in the tens place.

4) Finally he said the number 2 is sitting in the thousands place. What is the number?

Uncle Henry Problem read is read out by the teacher.
The teacher gives students copies of the problem and students work at the problem in their groups quietly. The teacher then proceeds to ask children to

explain their actions.

Teacher: Explain what you did.

Student X: I did units, tens, hundredths and thousandths and put a line underneath them. Then I read all through it again: 1, 2, 3, 4. I wrote down the number 2 in the thousands. In the hundreds place here I wrote 3 times the number in the thousands place, 2 multiplied by 3 which is 6. Then it says the number in the ones place is 4 times the number in the thousands place. So multiply one by 4 which is 4. This means 4 goes into the tens and one goes into the units. So that's 1462.

Teacher: Okay Student Y, explain to me what you did so.

Student Y: I read over all of the instructions and then I read over 1 2 3 and 4. Then, the 2 is definitely in the thousands place. Then it says he remembers the number in the hundreds place is three times the number in the thousands place. I multiplied 2 by three which is 6 so 2 is in the thousands place. The number in the hundreds place is 4 times the number in the thousands place so I multiplied 4 by 1 which is 4 so it is 1462.

Teacher: That's incorrect; you have one fundamental mistake.

Student Y: What!

Teacher: We will go over here to this boy.

He said the number 2 is sitting in the thousands place so that was actually there so I put 2 down. He said he remembered seeing a number 1 so I just kept that in my head. He said in the hundreds place he remembered seeing a number three times the number in the thousands place. So I did 2 by 3 which is 6 so I wrote down 6. Then he said the number in the ones place was 4 times the number in the tens place so I just wrote down 4 there and I thought the 1 was just the units at the end so I did that by 4 and I wrote down 4.

Teacher: No that's not correct. Anyone else got it figured out?

Student Z: He said the number in the ones place is 4 times the number in the tens place. Ok, The hundreds place he remembers the number is 3 times the number in the thousands place

Teacher: Okay, everyone listen now here. He said the number 2 is in the thousands place so 2 is in the thousands place. In the hundreds place he

remembers seeing three times the number in the thousands place so that is 2 multiplied by 3 which is 6.

Teacher: Where does that go?

Student Z: Under the hundreds. He said the number in the ones is 1 so we put the 1 there and the 4 in the tens.

Teacher: Why did you put the 4 there?

Student Z: Because you told me to, I don't know.

Teacher: Look at number 3. He said the number in the ones place is 4 times the number in the tens place so the answer is 2614 not 2641. Do you understand where that came from. You had yours backwards. Some people started with the units instead.

From the above, it is clear that Mike provided students with significant guidance and assistance as they engaged in the problem-solving process. Students in Mike's classroom do not use group work or group discussion in their solving of mathematical problems, and this is very evident from the above. Students have little discussion with one another and focus on solving the problem on an individual basis. The teacher requests that students to explain their answers, asking direct questions allowing little opportunity to facilitate the student in the solving of the problem themselves. This is clear from statements such as 'no that is not correct' and 'that's incorrect, you have one fundamental mistake'. The teacher does not facilitate students in the problem-solving process, but rather takes a very direct approach to the exploration of this problem with the children. It is clear that children have very little understanding of the problem in the concluding stages of the lesson, as Student Z reveals 'because you told me to, I don't know?' There is little evidence to suggest pupils engaged in developing a mathematical plan to solve this problem before proceeding on an individual basis.

Following student explanations of work, it is clear that some confusion regarding an appropriate answer remains. The teacher proceeds to explain the answer to the question but student confusion remains.

4.6.5.2 Problem 2

If Jane is older than Kim, Kim is older than Sean. Sean is younger than Jane and Rachel is older than Jane. Can you place the children in order from oldest to youngest?

Teacher: If Jane is older than Kim, Kim is older than Sean. Sean is younger than Jane and Rachel is older than Jane. List the people from oldest to youngest.

Student X: If it says at the start that Jane is older than Kim and Rachel is older than Jane …

Student Y: Do you want to go back on that and look at the youngest?

Student Z: Lets see if we can work it out and then we can talk about it later.

Students spend time working alone on the problem.

Teacher: Can you listen; Student Z is going to explain his answer.

Student Z: Jane is older than Kim and Kim is older than Sean and Sean is younger than Jane and Rachel is older than Jane.

Teacher: So who is the oldest then? Did you figure it out?

Student Y: Rachel

Teacher: Rachel, right very good.

Student Z: Jane?

Teacher: And then Jane

Teacher: Right OK, and then Kim

Student X: And then Sean

Teacher: So who is the youngest?

Teacher: So you can put Sean down at the bottom and Kim is older than Sean so Kim will come next and then Jane will come after that, right? And we build up to the oldest one, very good.

The above mathematical problem-solving situation is conducted similarly to the previous problem. However, it is a simple problem that can be solved quite quickly and, perhaps, is not significantly challenging for pupils. The teacher pays little attention to the problem solving procedure agreed by the cohort of

participants prior to their engagement with this topic. Students do not appear to reflect on the problem, to devise a strategy of solution, or to analyse any final solution they may have come to. This may be because individual students themselves may not have needed their group members assistance in developing a procedure for solving the problem due to the level of difficulty involved. The teacher plays a very active role in the mathematical deliberations of students, asking very specific questions and allowing little time for students to experiment with their own methods of solving the problem. From the conversation above it is clear that the teacher leads the discussion. The teacher concludes the problem- solving episode by stating the answer for the pupils as with to the previous problem.

4.6.5.3 Problem 3

Farmer Tom put a square fence around his vegetable garden to keep the deer from eating his corn. One side was 10m in length. If the posts were placed 2m apart, how many posts did he use?

Teacher: Firstly, read the question together.

Student A: Farmer Tom put a square fence around his vegetable garden to keep deer from eating his corn. One side is 10m in length. If posts were placed 2m apart, how many posts did he use?

Student B: So length is 10 here, so it is the same on the bottom as on the sides.

Teacher: Why have ye labelled the sides 2m? Shouldn't it be 10m?

Student A: Should we do area and all that; it might be right to do that.

Student B: You know the way it says posts, where are the posts, we don't know. They must be at the side or something.

Teacher: Look! It says the posts are going to be 2m apart around the edge.

The teacher draws and shows them.

Student A: And the posts, are they on the sides?

Teacher: That's what it says isn't it?

Student A: That's ten at the top, so 2 4 6 8.

Teacher: That's right.

Student A: So there are 5 across there.

Student B: Do we need to know area?

Teacher: Think of it. How can you figure out the length of this side now? Read the question; it is telling you how.

Student B: It's 10m

Teacher: Yes, how do you know that's that one?

Student A: It's a square so all sides are equal. They are all 10.

Teacher: Good, now start putting in your posts as if you are the farmer. Put in your first post. How many are you going to put down in the first line?

Student B: 1 2 3 4 5 on top.

Teacher: Ok, explain the top.

Student B: I'm marking them around the square; there are 5 on each line around   the   square.

Student A: How many did you get?

Student B: I don't know yet.

Student A: Did you get 5 on every single one?

Teacher: Do you think, be sure. I'm going to ask Mark to go to the board and explain his answer. I want everyone to listen to him then.

Teacher: Mark is after coming to the conclusion that the perimeter is 40 m in total. Where do you think he got that from?

Student A: Each side is 10 and 4 10's is 40.

Teacher: Good, the tricky bit was that he put a square fence around the field. What does square mean?

Student B: Each side is the same.

Teacher: So Mark told me each side is 10m he had to place the posts 2m apart. So in his copy he said he would start here. So here is where he put the first post right.

We put another at 2m, 4m, 6m, 8m, 10m, 12m, 14m, 16m, 18m, 20m, 22m, 24m, 26m, 28m, 30m, 32m, 34m, 36m and 38m. The one at 40 is there already so let's count to get our answer.

Students' work is displayed below.

Figure 10: Mike's students' work (Problem 3)
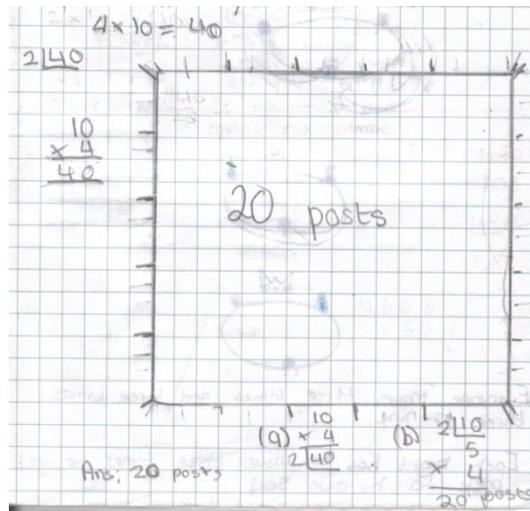
Mike begins this mathematical problem-solving episode by asking students to read the question together. This is in contrast with the actions of pupils during the previous episodes. Following this, however, the teacher continues to have significant interactions with pupils as they progress towards solving the mathematical problem. As students prepare a strategy to arrive at a solution, the teacher regularly provides them with direct instruction. The teacher proceeds to draw an outline example of what the farmer's field with the fence posts would look like. The teacher asks students specifically to 'start putting in your posts as if you were the farmer', having given the students a starting point. The teacher concludes the lesson in a similar way to other problem-solving lessons mentioned, by stating and explaining the answer clearly for students.

4.7 Conclusion

This chapter has revealed the case studies of Carmel, Emily, Joe, Tomás and Mike. It has charted their attempts to teach mathematical problem-solving from a constructivist perspective with their respective students. Their stories reveal the implications of constructivism for day-to-day classrooms, and have highlighted particular issues that must be addressed in relation to the employment of constructivist practices in the mathematics classroom. By engaging with the individual teachers in semi-structured interview, observing

their explorations of mathematical problem-solving from a constructivist perspective on site, and interacting with their pupils a comprehensive picture of every teacher's endeavours, practices, and beliefs has been painted. Chapter five examines in depth each individual participant, drawing on their teaching of mathematical problem-solving, their beliefs as revealed to the researcher, and the experiences of the students in the individual class, in a series of themes.

Chapter 5

Analysis of Findings

5.1 Introduction

This chapter presents an analysis of the cases on an individual basis. From the series of semi-structured interviews with participating teachers and group interviews with students, audio evidence, and documentary evidence a number of themes emerged that reveal participants' approaches to the employment of constructivist teaching practices in mathematical problem-solving lessons and place their approaches to mathematical problem-solving in context. Each participant's case is analysed on an individual basis in this chapter.

5.2 Participant one: Susan

Susan was a participating teacher in a Limerick City school.

5.2.1 Introduction

Susan is an energetic teacher, eager for her students to learn. Her school is a large suburban primary school where children come from middle class backgrounds and resources are plentiful. The school has very few difficulties in terms of parental support, resources or teaching space. Susan has experience, has a high level of education, and is committed to helping students achieve their best. Her teaching does reflect the principles of the Primary School Curriculum (1999), but she teaches for understanding and the structure of her classes discourages in-depth explorations of a student's understanding. By Susan's own admission, this is due to the various learning styles present in her classroom. Susan is eager for children to experience all strands and strand units of the mathematics curriculum but her particular situation, and the children that

she teaches this year, have required Susan, in her opinion, to take a traditional approach to the teaching of mathematics. It becomes evident that Susan's assumptions about learning based on the importance of memorisation and practice rather than on activity.

5.2.2 Susan's didactic teaching style

Susan identifies herself strongly as a good learner of mathematics. She also has a passion and enthusiasm for the subject and feels that it was her own teachers that fostered this. Susan has adopted a traditional approach to her teaching of mathematics and freely admits that this approach stems from her own experiences as both a primary student and secondary student. She recalls clearly her experience as a student of mathematics and believes that they have had a significant impact on how she teaches mathematics. Susan enjoyed her time at primary school and was challenged by her teachers through their use of difficult textbooks and mathematical problems. At primary level, Susan's teachers challenged brighter pupils by supplying them with mathematical problems from textbooks such as *Figure it Out* and *Busy at Maths.* Susan challenges her own high achieving mathematics pupils by supplementing their daily assignments with mathematical problems taken from other textbooks. She has great respect for her own teachers of mathematics and places significant importance on the methodologies and strategies utilised by those teachers. It is evident that Susan employs teaching methodologies similar to those she herself experienced as a pupil. She places significant emphasis on direct instruction. According to students, Susan spends a significant amount of time utilising direct instruction during mathematics classes: 'We correct our homework first and then she would explain something and ask us to do questions on it. She does things on the board loads of times and then we go and do it ourselves' (Appendix C.1).

5.2.3 Susan's focus on computation

Students in Susan's classes must have a significant understanding of and experience in the operations addition, subtraction, multiplication and division before progressing, for example, to exploring mathematical problems in association with their peers. In fact, students spend the majority of time in Susan's mathematics classroom working alone. Susan's students rarely spend time working together during typical daily mathematics lessons.

Susan is critical of the achievements of today's primary mathematics students when recalling the accomplishments of students of *Curaclam na Bunscoile* (1971). She attributes achievements of students of *Curaclam na Bunscoile* to methods of teaching mathematics that can be described as traditional. Children need to 'have copies where they repeat and repeat their sums' (Appendix C.1). In her analysis of constructivist teaching and learning Susan even went on to say: 'It is very valuable but it has to be used in conjunction with the rote learning, the chalk and talk and the teacher directed learning' (Appendix C.1).

Susan's mathematics classes were characterised by traditional conceptions of mathematics teaching and learning. She used new materials and ideas yet conducted exercises in a thoroughly traditional fashion. This is evident in the manner Susan conducted the mathematical problem-solving lessons with her students. Susan regularly uses direct instruction in her exploration of mathematical problem-solving by launching into an explanation of a problem before children have the opportunity to decide on an appropriate solution or strategy or to explain such strategy themselves. In the following instance, Susan asks a student to explain a solution to a problem, the student is hesitant in describing her solution to the problem. Therefore, Susan proceeds to explain the solution to the problem for the student to the rest of the class.

Teacher: Student X – will you explain to us please what you did.
Student X: I can't really remember by looking at this.
Teacher: What Student X is trying to say to us is that her group named the carriages 1, 2 and 3. You could get 1, 2, 3, you could get 1, 3, 2. Then you

might put number 2 first and get 2,1,3 2,3,1 3,1,2, and 3,2,1. They are all the different ways they can be arranged so let's count them – 6. I think most groups got that, Good job, well done.

Susan teaches mathematics for understanding and the didactic format of her teaching inhibits pupils own exploration or explanation of their ideas. This is due, significantly, to her experience of such methodologies at school, as she acknowledged, third level courses on methodology were less than informative.

Susan has a traditional view of what counts as mathematical prowess and her conviction about her approach was plain. Students in Susan's classroom study the foundations of operations, algorithms and procedures in significant detail. They spend significant amounts of time doing mathematical operations that are supplied by the teacher and by textbooks. Mathematical knowledge is broken into clearly defined units, particularly for those students who may be having difficulty with the subject. This became apparent as Susan explained that constructivist philosophy could only be employed in her particular classroom on a 'topic by topic' (Appendix C.1) basis. In fact, Susan explained: 'Forget problems that require a number of concepts or operations' (Appendix C.1).

5.2.4 Susan's emphasis on the rote memorisation of number facts

Such traditional conceptions of mathematics teaching and learning are clearly evident in her strong belief in the need for the rote memorisation of number facts or, as they are frequently referred to by Susan, tables. Susan has a strong conviction that students need to be fluent in the operations. When Susan was a student at primary school her teachers placed a significant emphasis on the rote memorisation of number facts or tables. The rote memorisation of such facts is at the heart of Susan's teaching of mathematics and Susan is unwavering in her belief that every student must have a comprehensive understanding of tables. Susan requires all of her students to memorise their tables and spends a portion of her allocated time for mathematics on the examination of these tables on a regular basis. During one visit to Susan's classroom, Susan had spent 15 minutes examining tables before proceeding to explore a mathematical problem.

She explained in interview: 'We need to go back a little bit to the old style where it was drill' (Appendix C.1). Susan's conviction is such that her students do not progress very far beyond simple computation and simple problem-solving if they have not a comprehensive knowledge of their tables. She explains that this is particularly the case in this academic year because she regards her students as mathematically weak. Susan had solid reasoning behind her view that students must have a comprehensive understanding of tables if they are to succeed in mathematics; she highlighted the fact that students will need to have a capacity to do mental mathematics in nearly all avenues of life including, as she outlined, simple shopping expeditions. It is here she warns of overusing the calculator at primary level. Susan is convinced children can become over reliant on the calculator and consequently do do not develop the ability to become proficient at mental mathematics.

5.2.5 Susan's difficulty with a high pupil teacher ratio

At the outset of the semi-structured interview Susan asked if she could be controversial. One issue that Susan feels very strongly about is the lack of resources and procedures that enable teachers to teach from a constructivist perspective in the Irish primary school context. Susan speaks from her own perspective and, in particular, feels very strongly that the pupil teacher ratio in her school (something she shares with colleagues in other schools) prevent the employment of constructivist methodologies in her teaching. The current ratio is 28:1, but Susan explains that, in reality, this is always greater when numbers of teachers in Learning-Support and Language Support are taken into account. Observing Susan's classroom, it is clear that exploring mathematical problem-solving in group situations is difficult because of the limited space available. The population of the area where Susan teaches has risen sharply in recent years and this is reflected in the number of pupils enrolled at the school. Susan feels that this inhibits her in approaching mathematical problem-solving from a constructivist perspective.

Susan finds it difficult to explore basic concepts with students of varying abilities in her mathematics classes because of the numbers of students

involved. During a typical lesson Susan endeavours to teach a particular concept to all students through direct instruction, and then segregates the class into groups, assigning work to one group as she continues to teach another who may have difficulties in understanding. She explains it is 'next to impossible' to teach that many children. Susan cannot see how one would have children solve problems from a constructivist perspective on regular basis given the difficulties that arise during classes involving direct instruction with large groups of students.

5.2.6 Susan's use of group work

Susan's groups were used for instructional purposes in a distinctive way. During lessons, Susan directed the students to answer questions related to a problem rather than facilitating them in their quests to identify questions and problems related to the construction of an appropriate problem-solving method. Susan was using a traditional approach to the teaching of mathematics while combining a reform based approach to facilitate her teaching. This is clearly evident in the following.

Teacher: But would they add up to 99 if you used 8 addition signs? $9 + 8$ is 17 plus 7 is 24 plus 6 is 30 plus 5 is 35 plus 4 is $39 + 3$ is 42 plus 2 is 44 and plus 1 is 45. So no, try and put some of the numbers together.

Student B: What about $1 + 2 + 4$?

Student C: But it will still give you 45/

Student A: We have to be careful of how we use the big numbers.

Teacher: How about joining your 7 and 6 together maybe?

Student A: you put the 2 and the 1 together that's 21 and 9 is 30.

Student B: there are lots of ways; the 3 and the 5 together is 35.

Student A: We have to be careful and keep the 9 and the 8 separate – careful how we use the big numbers.

Student B: Are we allowed move around the numbers or do they have to be like that?

Teacher: Start at the beginning and work it out.

Susan gives students opportunities to interact with each other but is very clearly directing students towards a strategy of solution clearly identified by the teacher in advance.

5.2.7 Susan's approach to teaching pupils with different learning abilities

Susan teaches sixth class students identified as mathematically weak by the school using the Drumcondra Primacy Mathematics Test. This is a standardised test published by the Educational Research Centre in Drumcondra, Dublin. Susan describes her particular situation as 'different from others' (Appendix C.1). Susan explained that, as her students did not have a firm understanding of operations, exploring mathematical problem-solving from a constructivist perspective was going to be particularly challenging for them from the outset. From the initial stages of the project until its conclusion Susan believed that it is unrealistic to approach mathematics from constructivist perspective with low achieving students of mathematics and this restricted her students from experiencing the approach to the subject discussed during professional development. This is evident in her explorations of particular problems with students. Susan offers students the solutions and strategies of solutions to the problems giving students little opportunity to reach a conclusion or design an appropriate problem-solving strategy themselves. Susan's lessons were regularly brought to an abrupt conclusion by her exploration of the problem at the blackboard in front of the whole class. This is illustrated in the following.

Student A: This is complicated we need the teacher.

Teacher: Can I give you a hint; some people have worked out that if there are 140 eyes in total there are 70 animals altogether as each animal has 2 eyes.

Students A, B and C: Oh, 70 animals

Teacher: So now we have to figure out all the different ways of making 70 and see which would make sense. Take a guess, 30 chickens, so 2 legs each is 60 legs and then there would be 40 pigs and 4 by 40 is 160 – so it is 230, could that be right?

Class: No

Teacher: It's all trial and error that's what we have to do, make guesses and check them out.

Teacher: Don't rub out any of your answers. Remember the eyes are sorted and that it is the legs that we need to work on. Have we an answer?

Student A: 30 pigs and 40 chickens!

30 pigs and 40 chickens have 140 eyes so that is right

30 pigs will have 120 legs and 40 chickens will have 80 legs.

Teacher: That is 200 legs altogether, that's right, well done.

Susan's belief in the mathematical abilities of the children in her care is highlighted in the following quotations. They also explain Susan's significant involvement in the students' attempts to construct a strategy to solve the problem:

> I think to be honest because they were particularly weak, having an idea or putting an idea about something forward would have caused difficulty in any subject area not to mind maths. They need the teacher as a crutch. They couldn't even put an argument together in English, one sentence and that was it (Appendix C.1).

Susan is open to students solving problem solving from a constructivist perspective but students must have particularly strong background knowledge of mathematical concepts and operations, as illustrated by the following:

> Some of them have even difficulties adding hundreds tens and units and some of them had some idea about for example the addition of fractions so a very mixed bag indeed. Constructivism is great and I will do it next year where I know my class will enjoy it more and get more benefit out of it but this year is particularly hard (Appendix C.1).

Susan continued to elaborate explaining: 'I can really see it working well with more able students.

Susan described the interpersonal skills of the students as weak. According to Susan, these students did not have the required skills to work appropriately in group situations. Throughout their primary school years, due to a poor understanding of basic mathematical concepts, this group of children, according to Susan, had very little experience of mathematical problem-solving. Susan explains: 'It was a lack of problem-solving, they hadn't experienced enough of it, but where do you go if you can't add, subtract or multiply?' (Appendix C.1). Significantly, Susan believed that her students had particular difficulties with memory, explaining that students, when they returned from a break or holiday period, would act like they had never seen the material before. This would suggest Susan would have to revert to exploring basic number facts and operations with students repeatedly. These students were at a disadvantage; because teachers consistently labelled them as mathematically weak they experienced little, if any, constructivist approaches to mathematics in the later years of their primary schooling.

5.2.8 Susan's constructivist approach to learning

Susan's classroom was organised for co-operative learning but her instructional strategies cut across the grain of this organisation. The class was conducted in a highly structured and classically teacher centred fashion, as illustrated in the mathematical problem solving episodes. Susan has considerable experience as a teacher and also has experience as a teacher educator and has, therefore, a sound understanding of the implications of approaching mathematical problem-solving from a constructivist perspective. Her reservations about exploring mathematics from such a perspective have not arisen from a lack of understanding of constructivist theory but rather from reservations as to its appropriateness in the particular situation. This is clearly evident from the fact that Susan acknowledges the value and purpose of a constructivist approach to learning but then said: 'It has to be used in conjunction with the rote learning, the chalk and talk and the teacher directed learning' (Appendix C.1). Also, the mathematical

abilities of students are taken into account before Susan employs particular methodologies in her lessons.

She describes a constructivist approach to teaching as 'about problem-solving, finding out where the students are at and then building upon it' (Appendix C.1). She continues: 'It's about giving a little bit more ownership to the students. It is going away from directed learning' (Appendix C.1). This is in line with a constructivist approach to learning. She admits, however, that 'more guidance is required' (Appendix C.1) and that 'the material isn't there to facilitate the teacher' (Appendix C.1). Susan believes the vast majority of teachers would not approach the teaching of mathematical problem-solving from a constructivist perspective due to a lack of pedagogical knowledge: 'It may say it in the curriculum, but I don't think many teachers would be familiar with how to go about doing it in the classroom' (Appendix C.1). This reinforces a finding by the Primary Curriculum Review: Phase 2 (NCCA, 2008a), which found that teachers are challenged in developing a child's higher level thinking skills and that whole class teaching strategies are the most frequently used teaching strategies in primary classrooms.

5.3 Participant two: Emily

Emily was a participating teacher teaching in a Limerick city school.

5.3.1 Introduction

Emily is a dedicated teacher with over thirty years experience in the classroom. She is very committed to her profession and it became apparent throughout the study that she is very eager for all students in her care to achieve to the best of their ability. She worries about her students achievements, she feels uneasy if students have not experienced, in her opinion, what she believes is appropriate instruction in mathematics. Her school is a large suburban primary school where vast quantities of resources are at her disposal for the exploration of the Primary Curriculum (1999) as it was envisaged.

Emily is particularly traditional in her approach to the implementation of the Primary Curriculum (1999) in her classroom. Emily acknowledged that she focuses on English, Irish and Mathematics with her students and finds opportunities to 'squeeze in' (Appendix D.1) other areas of the curriculum. 'What I'm finding is I get the major four out of the way (English, Irish, Mathematics and Religion) and then spend a day or two doing the other subjects like history, geography, etc ' (Appendix D.1).

From working with Emily, it is clear that she is open to engaging in professional development and she is eager to endeavour to incorporate different approaches into her daily classroom routine. This was evident in her enthusiasm shown for this project form the outset. Although Emily may be described as traditional in her approach to teaching, it is her commitment to her students and her openness to innovation that led her to embrace a constructivist approach to the teaching of mathematics.

5.3.2 Emily's didactic teaching style

Emily freely describes herself as an average student of mathematics but has a keen interest in the teaching of the subject as, in her opinion; it is an important

subject area of the curriculum. Emily's own teachers were very traditional in their approach to the topic and she acknowledges that there was some merit to the methodologies and topics approached by these teachers. She particularly remembers primary teachers who placed significant emphasis on mental mathematics and this emphasis can be seen today also in Emily's own class teaching, Emily begins every lesson with a mental mathematics session. Interestingly, although corporal punishment was employed during Emily's own primary schooling during mental mathematics lessons, Emily remembers fondly such lessons. Emily admits that during second level, teachers who did not have comprehensive discipline knowledge did not engender a love of the subject amongst students.

Emily likes to remain loyal to her tried and tested methods of teaching mathematics; she describes her own teaching as 'structured' (Appendix D.1). She acknowledges that she experiments with teaching methods that might be new to her, such as group teaching, but that 'I like to stick to old fashioned methodologies' (Appendix D.1). Emily will explore new methodologies only when she has a comprehensive understanding of how to utilise them effectively in the classroom. This became apparent as Emily explained during the initial stages of the project that, until she had seen a lesson conducted from a constructivist perspective she would not have felt comfortable in designing and conducting such a lesson. This was significant given that Emily had taken part in discussion and professional development designed around the theme of constructivism in advance. This implies that successful professional development initiatives must involve the integration of the participants' classroom and students.

Emily employs a didactic approach to the teaching of mathematics. The structure of her classroom is such that children sit in rows facing the teacher and are allocated places according to their achievements in the Drumcondra Primary Mathematics Test. Emily had not explored mathematics through group teaching prior to her engagement in this exercise. Children are surrounded by the number facts and the outlines of basic procedures for performing the operations addition, subtraction, multiplication and division; they are clearly displayed on

the walls of the classroom. The significant focus of Emily's mathematics classroom teaching for the academic year is number. Emily admits 'I like to stick with the nitty gritty, number, fractions, decimals and percentages and the like' (Appendix D.1) she explains 'I then move on to the more light hearted areas a I like to call them, of data, chance and length' (Appendix D.1). She reveals 'I give 60:40 to the number strand compared to everything else' (Appendix D.1). Throughout the period of research, Emily remained faithful to her approach to the teaching of mathematics by ensuring children received instruction in the subject whilst engaging in this study, mathematical problem solving from a constructivist perspective, during discretionary curricular time.

5.3.3 Emily's emphasis on the rote memorisation of number facts

Emily employs traditional teaching strategies in her mathematics classroom. Receiving particular attention in Emily's classroom include pencil and paper computations, rote practice, rote memorisation of rules, teaching by telling, and the memorisation of facts and relationships. Student descriptions of a typical mathematical lesson in Emily's classroom support this 'we start with mental maths, twenty questions and we have five minutes to do them, and then we move on to our *Mathemagic* for ages' (Appendix D.2). By requiring students to write 'all multiples to 15' (Appendix D.1) in their copybooks numerous times, it is clear Emily places significant importance on rote memorisation. This does stem from her observations of teachers during her time as a student as discussed earlier.

5.3.4 Emily's constructivist approach to teaching and learning

Emily can describe a constructivist approach to learning. 'It is putting an interesting task on paper in front of children and getting them in a group to solve the problem. It's not showing them how to get to the answer but directing them if needed, popping questions out there to make them think in the right direction is useful' (Appendix D.1). She embraced the teaching of a mathematical problem solving lesson from a constructivist perspective with vigour. This is clearly evident in her approach to facilitating students as they

solved mathematical problems chosen by her. Emily's input during all lessons was clearly constructivist. Once Emily had seen a mathematics lesson conducted from a constructivist perspective, she was quick to adapt and achieve success in her own constructivist endeavours. Emily asked children to reflect on problems, to 'tell me quickly the important information and maybe also tell me some unimportant information' (Appendix D.1), to 'explain for me your answer' (Appendix D.1) and also encouraged students to write written explanations of their solutions to the problems. She rigidly encouraged students to follow Polya's (1971) four stage problem solving procedure. This is evident form the student's written work.

Emily likes routine and her constructivist lessons followed clearly routine established while the teachers were engaged in professional development. Emily asked students to keep a record of solution strategies that they might have initially chosen to solve the problem, but amended and adapted as they required, and also to comment on their solutions. She found this useful in follow up discussions based on the problems.

Figure 14: An example of Emily's students' explanations
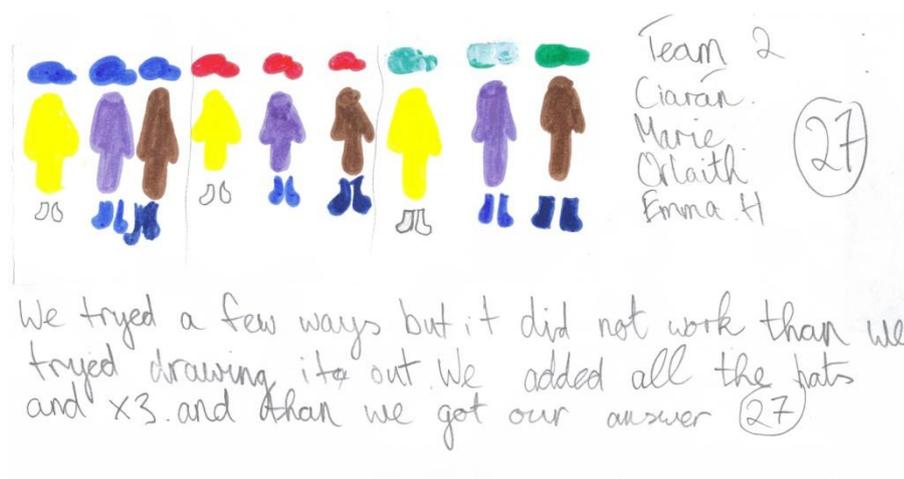
Students also found these discussions very useful as one student emphasised 'explaining something out loud helps you understand' (Appendix D.2). She discussed these strategies in conjunction with their successful strategies following the lesson. At all stages, Emily assumed the role of facilitator asking

probing questions of students in difficulty. The following lesson is an example of Emily's interpretation of a constructivist approach to problem solving and, as highlighted above, Emily's interactions are clearly constructivist.

King Arnold sits at a Round Table. There are three empty seats. How many ways can 3 knights sit in them?

Student A: Reads the problem,

Teacher: Firstly, think, have I done something like this before, is there a method I have used before that might be useful

Student A and B: Let's draw a round table with chairs, 4 chairs, empty ones.

Student A: Ok now what do we do?

Student B: Let's read it again aloud together.

Student A: 3 empty seats so and King Arnold in one of them.

Student B: Let's draw him.

Student C: What do they mean about 'how many different ways?'/

Teacher: Can anyone tell me quickly the important information and maybe also tell me some information that is not important.

Student C: The name of the King

Student B: The round table and the 4 chairs are important.

Student A, B and C: This is very hard, let's read it again and again

Students read problem

Student B: Maybe it is kind of like the doll problem – one of them might sit her then move to here and then to here.

Student C: And then, that one swaps and sits in the other seat.

Student A: So draw one table and draw a crown at the top. Then we will call them King 1, King 2 and King 3.

Student B: Why King 1.

Student A: Ok, Knight 1, Knight 2 and Knight3.

Student C: Yes, Knight 1 can be pink, Knight 2 can be blue and Knight 3 can be green. Now we will move them around.

Student A: Yeah, now Knight 1 moves to Knight 2 seat, Knight 2 to Knight 3 seat and then Knight 3 moves to Knight 1 seat.

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Student C: So that is nine? Is it?

Teacher: Can you explain for me your answer

Student B: We drew a round table and the king was on top and his crown was in yellow. Then we had King 1 at the first chair, K2 at the second chair and K3 at the third chair.

Student B: Then we moved K1 to K2 chair then K2 to K3 then K3 to K1 chair.

Teacher: Good.

Student C: Then we did it all over again

  K1 went to K3

  K3 went to K2

  K2 went to K1

Teacher: Make sure you write an explanation.

Student A: For the second part of the sum we drew another round table. This time there was 4 empty seats.

Teacher concludes by explaining the question to the class.

From reminding students of the steps to follow in approaching a mathematical problem to encouraging students to explain their choice of strategy and asking them to write explanations for their answers, Emily succeeded in teaching students from a constructivist perspective during this episode. Students were capable in selecting an appropriate strategy for solving the problem and explained this to the teacher well. Interestingly, Emily explained following this lesson that students often presented strategies that she would not have designed herself. Emily also explained 'I am learning with the children as it is so new. When we are all learning like that, including me, it brings excitement into the room and they want more of it' (Appendix D.1).

From evidence gathered from students, it is clear Emily put significant effort into ensuring a constructivist experience. Students described some details of their activities during the lessons.   The following also illustrates Emily's employment of Polya's (1971) four stage problem solving procedure.

- 'We go through it for a plan and then talk about what we think about it before we go and do anything' (Appendix D.2)

- 'We would talk about what's useful and what isn't' (Appendix D.2).
- 'We have a highlighter for the important things so you don't have to read over the whole problem again' (Appendix D.2)
- 'Sometimes we just talk and don't do anything' (Appendix D.2).

Students explained that their teacher would encourage them using phrases like 'You're almost there' (Appendix D.2) on a regular basis.

5.3.5 Emily's approach to teaching pupils with different learning abilities

Although Emily describes approaching mathematical problem solving from a constructivist perspective as valuable, she has great concern for students with learning difficulties in relation to mathematics. Emily found that mathematically capable students tended to dominate group work. She explained 'they were missing out on something special' (Appendix D.1). Research into engaging all students in successful group exercises has shown that if students do not work in group situations and become clear on the roles and responsibilities of everyone in that group that stronger characters will dominate (NCCA, 2006). Students in Emily's classroom were rarely exposed to group work and this may account for her concerns. Emily explained that she would alter the structure of the individual groups according to ability and provide students with specific learning difficulties 'extremely simple problems and a lot of guidance' (Appendix D.1).

5.3.6 Emily's students' engagement with mathematical problem solving

Student interest and excitement in approaching problem solving from a constructivist perspective was obvious and acknowledged by both students and teacher. Emily explained that it was difficult to restrict their explorations of mathematical problems to the time allocated in the classroom. Students had a strong appetite for the problems that were being explored and in particular for the constructivist approach employed to solve these problems. One particular student, when asked to comment about the lesson indicated a significant interpretation about solving mathematical problems in co-operative group

situations. He explained 'It was more about how you got the answer than the answer, how you worked it out' (Appendix D.2). As discussed, the focus of Emily's regular mathematics class is traditional, students found an alternative approach to solving interesting as 'you could do it in groups and that was more fun. It wasn't all about getting top marks in the room, it was just about the one question at the time' (Appendix D.2).

Students were motivated by the 'more interesting than normal problems' (Appendix D.1). Students explained that working on mathematical problems in group situations was appropriate for them because of the opportunities it provided for them to interact with their peers and that assistance was available quickly from peers. Students delighted in the opportunity to decide for themselves what materials or strategy was appropriate to the situation. They also revealed 'you really know how to explain it to someone afterwards because we might have different ways of solving the same problem' (Appendix D.2).

Students developed a logical strategy for solving unfamiliar mathematical problems encouraged by Emily. Students explained that it is helpful if initially, everyone records their thoughts on the mathematical problem before progressing to the planning stage. Students were quick to highlight the need to break the problem up so that 'you can do it in little steps' (Appendix D.2). Students explained that solving such problems in a constructivist learning situation was 'hard in a good way' (Appendix D.2). This stemmed from their enthusiasm for working together in cooperative group situations and from their interest in the subject of the mathematics problems. In the end, students explained that solving an unfamiliar problem 'took a little longer that's all' (Appendix D.2).

5.4 Participant Three: Joe

Joe was a participating teacher in a Limerick City school

5.4.1 Introduction

Joe is an experienced class teacher and has spent the majority of his career to date teaching students at 5th and 6th class level. Joe has a significant interest in the teaching of mathematics stemming from his belief in his own mathematical ability. Joe recognises that students require a 'good knowledge' (Appendix E.1) of maths in every avenue of life and he is therefore eager that students would enjoy studying mathematics. Joe described himself as eager to undertake such an initiative as he enjoys bringing 'variety and challenge' (Appendix E.1) to the subject. Joe is particularly interested in helping students become able problem solvers. Joe believes he achieves this by supplementing work assigned to students from textbooks with material he has gathered throughout his teaching career. He lists old textbooks, including *Figure it Out*, *Busy at Maths* and *Maths Challenge,* newspaper cuttings, including mathematical quizzes published by newspapers and reference books as the sources of this supplementary material. Joe places significant emphasis on the utilisation of textbooks in the mathematics class. Within his classroom, there is a variety of mathematics textbooks gathered from when Joe began teaching, in particular early editions of *Figure it Out*, and he utilises almost all of these books on a regular basis. Joe describes the introduction of the Primary Curriculum (1999) as the biggest challenge faced by him during his teaching career.

Joe describes the Primary Mathematics Curriculum (1999) as 'very much focussed on the full participation of the child' (Appendix E.1) that 'students understand what they are at' (Appendix E.1). Interestingly, Joe highlights that primary teachers have become complacent in teaching mathematics at this level. He explained 'a focus on computation resulted in children having little or no understanding of the actual concept behind the mathematics' (Appendix E.1).

During the following analysis it becomes clear that Joe utilises traditional methods for teaching mathematics with his students. Joe was extremely interested and eager about constructivist practices and their implications for the mathematics classroom given his very significant interest in mathematics. Yet, it will be shown that Joe's actions and instructions to pupils as they engaged in mathematical problem solving from a constructivist perspective reveal that he found it difficult to fully implement a constructivist approach to mathematics teaching.

5.4.2 Joe's traditional approach to the teaching of mathematics

Joe describes pencil and paper computations, rote practice, rote memorisation of rules, teaching by telling, and the memorisation of facts and relationships as having little effect as compared to children actively engaging with the subject and in particular, using concrete materials. Joe described that before the publication of the Primary Curriculum (1999), he engaged students with their environment in an effort to take mathematics outside of the classroom. Joe revealed in detail a particular lesson on area and perimeter and the use of the local Gaelic Athletic Association football field as an example. This example however was not one of activity in the recent past, nor is it something that is repeated on a regular basis. Joe explained that the mathematical problems students appeared most enthusiastic about, that he chose for exploration from a constructivist perspective, were the problems that 'took them out of the classroom and into the environment' (Appendix E.1). Joe revealed that although he recognises the importance of bringing mathematics outside of the classroom it can be difficult to organise and manage on a very regular basis so he admitted that it therefore does not happen on a very regular basis.

For their mathematics lessons, Joe's students engaged with each other collaboratively only occasionally before their involvement in this research. Joe explains that due to the large sizes of the classes he has to teach, they can often be difficult to manage and, more often than not, students sit in regular rows facing the teacher engaged in direct instruction with the teacher. Joe's students

reveal that 'when we get lots from the book and I can't do it, I really hate it' (Appendix E.2). Joe's students describe typical lessons 'we take out a book, if we are starting something new, he will do a sum and then we go off by ourselves and do it' (Appendix E.2).

5.4.3 Joe's belief in constructivist experiences as enrichment activity

The Primary Curriculum (Government of Ireland, 1999a;1999b), in Joe's opinion, gives teachers the opportunity to work with students who may be regarded as mathematically weak. He explains 'it is more child friendly' (Appendix E.1) and the 'problems and topics are not as difficult as they were, for example 10 to 15 years ago' (Appendix E.1). For capable mathematics students, Joe explains, the teacher must have the ability to think beyond the curricular documents and plan for lessons to meet the needs of such students. Joe reveals 'you can challenge a good class if you put your mind to it' (Appendix E.1). Joe identified a constructivist approach to mathematical problem solving as suitable challenge for capable mathematics students. It became clear however, that Joe does not see identical benefits for students who may have learning difficulties in mathematics. Joe has concerns for the students who were grouped according to mixed ability.

Joe revealed that whilst engaging the students in mathematical problem solving from a constructivist perspective, he felt students with learning difficulties were being dominated by more capable mathematics students. Joe went on to describe approaching problem solving from such a perspective as suitable to those 'well able to read and decipher a situation' (Appendix E.1). In contrast however, Joe explained that a student with 'flair' (Appendix E.1) for mathematics can be valuable in a co-operative learning situation. He said: 'The presence of a pupil who has a flair for the subject can be a big help. He can bring the rest of the students with him' (Appendix E.1). Joe noticed this as students engaged with each other.

5.4.4 Joe's understanding of a constructivist approach to mathematics

Joe has a clear understanding of constructivist approaches to mathematics education. He describes successful constructivist approaches to mathematical problem-solving as involving 'hands on activities' (Appendix E.1), the incorporation of the local and school environment, and 'ensuring a concept does not remain abstract' (Appendix E.1). Joe had this understanding before engaging with the research. He feels a student is at a greater advantage if he or she can come to an understanding of a concept under the guidance of a teacher rather than through direct instruction. However, it is clear from Joe's interaction with students during problem solving that his students are not afforded the opportunities to interact with one another for any length of time when they find themselves struggling with a problem. Joe provides them with significant assistance and advice rather than facilitating them through purposeful questioning. Joe indicated during interview: 'Spoon feeding them a concept or a skill really spoils a learning opportunity. They must be left to tease it out themselves, go as far as they possible can, and then provide guidance where it is required' (Appendix E.1). Joe provided a significant amount of this guidance as evidenced by the problem-solving episodes.

Joe describes a good mathematics student as one who can deal with a wide range of mathematical problems and one who can work out his/her own mathematical problem-solving strategies to solve them. Joe likes to witness a student 'going from the known to the unknown' (Appendix E.1). He explained that he limits the number of problems involving individual concepts, but, rather, presents them with problems that 'draw on their knowledge of various concepts' (Appendix E.1) so that 'children can use bits of knowledge simultaneously' (Appendix E.1). Joe identifies a good mathematics student as one 'who can deal with a wide range of problems and work out appropriate strategies to solve them' (Appendix E.1). However, from the problem-solving lessons, it was evident that Joe did not provide them the opportunity to grapple with such issues, but provided them with significant amounts of assistance.

5.4.5 Joe's didactic approach to mathematical problem solving

The following mathematical problem solving-lesson illustrates Joe's didactic approach to the teaching of mathematical problem-solving. It is clear from the interactions between students, and in discussion with them in group interview, that group activity is not a methodology employed in Joe's classroom, apart from the lessons conducted for the purposes of this research. Joe begins the lesson by reminding children to 'jot down quickly what information you might need to solve the problem, recall problems that you have done that were similar to these and anything that comes in to your head about the problem' (Appendix E.1). However, students work quietly alone and have difficulty in interacting with each other to find a solution to the problem. This is clear in the example below. Although Joe indicated throughout the research that students benefit from engaging with each other and solving problems without the significant assistance of the teacher, he does provide significant assistance and closely monitors the work of his students. The following mathematical problem-solving episode and, particularly, Joe's interventions reveal this.

Mark (from Sydney, Australia) and Hans (from Berlin, Germany) often communicate with each other using 'chat' on the Internet. They have to log on to the Internet at the same time to be able to chat. To find a suitable time to chat, Mark looked up a chart of world times and found the following:

At 7 p. m. in Sydney, what time is it in Berlin?

Greenwich 12 midnight

Berlin 1 a. m.

Sydney 10 a. m

Mark and Hans are not able to chat between 9 a. m. and 4:30 p. m., their local times, as they have to go to school. Also, from 11 p. m. till 7 a. m., their local times, they won't be able to chat because they will be sleeping. When would be a good time for Mark and Hans to chat?

Teacher: After you read the problem just jot down quickly what information you might need to solve the problem, problems that you have done that were similar to these, and anything that comes into your head about the problem.

Teacher explains the location of Greenwich and its relevance to different time zones using a wall chart.

Teacher: Has anyone experience of time difference from their summer holidays?

Student A: Yes, when I go it's usually an hour ahead of Ireland.

Students spend silently figuring out the problem on their own initially (8 minutes).

Teacher: My first idea would be to draw two clocks and colour in the times they would not be able to talk to one another, and look at and compare the clocks to figure out when might be a good time to talk.

Students proceed to do this silently.

Teacher: Now when Mark comes home from school at 4.30 in the evening, what time is it for Hans?

Student A: Nine hours difference.

Student B: He couldn't talk; no one would answer it would be 1.30 in the morning.

Teacher: Is Berlin before or after Sydney?

Student A: After, no before

Teacher: So when Mark gets home from school, what is Hans doing?

Student B: it would be 1.30 so he would be asleep.

Teacher: But before means going back.

Student C: So it is 7 o'clock.

Teacher: Would he be able to chat at 7.30.

Student B: Yes

Teacher: What about 5.30 p. m. Australian time.

Student A: That would be 8.30 a. m. Berlin, a good time.

Teacher: So between what hours would be good for them?

Students A, B and C: *Quiet discussion*

Student A: From 7.30 to 8.30 in Berlin and from 4.30 to 5.30 in Australia.

Teacher: Why did ye find that difficult?

Student A: We had never done a problem like it before. We mixed up going ahead in times rather than going backwards.

The above example is a clear description of how Joe interacted with his students and employed constructivist methodology. It highlights the lack of co-operative problem solving experienced by pupils and the significant assistance and scaffolding provided for them by their teacher. Joe felt that student participants in his classroom 'got a certain amount of satisfaction out of being able to solve a number of these problems that they originally thought they would not be able for' (Appendix E.1).   From the research it appears that it was in close interactions with the teacher that they achieved success.

During lessons, Joe began to engage students in problem-solving by following Polya's (1945) four stage problem-solving procedure with which teachers engaged during professional development. As evidenced in his interactions with groups of students, Joe spent time explaining difficult parts of the problems to students giving them significant guidance frequently. This is illustrated in the above example.   Joe did not scaffold the amount of assistance required by the student during the problem-solving episodes.

5.4.6 Joe's conclusion of a mathematical problem-solving lesson

Students were working at finding a solution to a problem and were progressing through the problem and their chosen method of solution when Joe asked another group of students to explain the solution to the class.

Teacher: 'Student X' is going to explain to the group.
Student A: What, but we nearly have it.
Teacher: Ok, but just listen to 'Student X'.
Teacher: 'Student X' has made sure that 10, 9, 3 and 4 are in one part of the clock and they add up to 26. She made sure that 11, 12, 1 and 2 are in another part, they also add up to 26 and then 8, 7, 6 and 5 are left and they make 26.
Student A, B and C: Ok so

In addition, Joe proceeded to explain the answer to the problem himself. Although Joe had conducted a thorough mathematical problem-solving lesson from a constructivist perspective, it appears the period of time required by particular students to solve problems may have caused Joe to draw the lesson to an abrupt conclusion and inhibit students in coming to a result on their own.

5.4.7 Joe's employment of collaborative group work methodology

Students in Joe's classroom explain that they work alone on mathematical problems on a regular basis. Students reported this as problematic because 'you can't talk about what you don't know so then you get stuck very easily' (Appendix E.2). Students reported an increased amount of success in situations where they interacted with their peers: ' Its a lot easier because … you can ask for help with a group or decide on what to do with other people and that helps you out too' (Appendix E.2). One student said: 'the problems are hard but it's easier when you are in groups' (Appendix E.2).

At regular intervals throughout the lessons Joe gives significant advice and support to his students. Examples from the above include: 'Now when Mark comes home from school at 4.30 in the evening, what time is it for Hans?' and

'But before means going back'. This indicates that students had not the opportunities to develop relationships with one another, during the research period, in a structured environment and therefore become accustomed to working together towards a shared goal. Joe freely admitted that it took time for students to adapt to the format of lessons. Clearly, students found it difficult to communicate their ideas with one another and structure a solution to the mathematics problem on this occasion. Students explained during interview that they spend a lot of time at work with textbooks alone and this may explain the difficulties these students had in interacting with each other, sharing ideas, and collaborating with one another during problem solving. Students were not skilled in collaborating with one another.

Joe decided that by grouping students according to ability every student could potentially perform to the best of his/her ability. Joe explained: 'Children have to feel that they have something to contribute and be able to follow reasoning that is going on' (Appendix E.1). Joe continued to leave students in mixed ability group situations for the duration of the research but remained concerned for the learning of these students as they engaged with their peers. It is clear from Joe's engagement with the pupils during their mathematical problem-solving that he provided significant assistance when students were struggling to develop a strategy for solution. This indicates the level of responsibility that Joe was willing to afford his students in their interactions with one another.

5.5 Participant Four: Tomás

Tomás was a participating teacher in a Limerick City school.

5.5.1 Introduction

Tomás is an enthusiastic teacher, a recent graduate who believes he employs a variety of current relevant methodologies in his teaching. He teaches fourth class students in a large primary school with thirty three-students in his class. Tomás describes his class as challenging, revealing the varied learning styles and abilities brought to the classroom by his students. Tomás pursued a career in media and communications prior to taking up his current position and admits that this has influenced his teaching.

Tomás enjoys using the environment, particularly the school and local environment, in his teaching. He likes to see children making connections with aspects of their locality that is already very familiar to them. Tomás is very interested in sport and this was evident in his utilisation of a variety of sporting analogies in his teaching. Tomás places a lot of emphasis on teaching for understanding and employs discussion and debate regularly in his teaching of mathematics. Tomás explained that children often 'discuss different methods you could use to come up with an answer' (Appendix F.1).

From the outset, students in Tomás' classroom had little difficulty in engaging in productive co-operative problem-solving; this was evidenced in the mathematical problems that the students engaged in. Interestingly, Tomás explained that, as a recent graduate of a college of education, he was very familiar with group teaching practices. Tomás is determined that children in his classroom will experience problem-solving as a topic in it's own right, revealing during the semi-structured interview that engaging in this research made him realise the need to teach mathematical problem-solving skills to students.

5.5.2 Tomás' problem-solving approach to teaching and learning

Tomás doesn't identify himself as being a good mathematics student, explaining that he has an average ability in relation to the subject. Recalling his experiences at primary school, Tomás explains that the only teacher who provided a memorable mathematical experience for him was 'a fourth class teacher who had us consistently problem-solving' (Appendix F.1). Interestingly, even though Tomás believes he has an average ability in relation to the subject, he said: 'I was always first up to the teacher with the answer' (Appendix F.1). Although not confident in his mathematical ability, his interest and enthusiasm for mathematical problem solving was apparent. For Tomás, his fourth class teacher stood out in particular because 'other teachers never really went beyond the pages of the textbook with the exception of this class teacher that I mentioned' (Appendix F.1). He explains that 'her lessons were very much what we are encouraged to do now in terms of using resources and having children solve problems' (Appendix F.1). Tomás endeavours to provide similar experiences for his students. Tomás believes his teachers at second level ensured that mathematics were more enjoyable for him because 'their methods were more refined' (Appendix F.1).

Tomás employs a problem-solving approach in his teaching of mathematics. He explains that before exploring a concept with children he encourages them to be open to trying out different methods or strategies in their solving of mathematical problems. This became evident as Tomás engaged with the children during the mathematical problem-solving episodes. He said: 'One of the fundamentals is that, when we are starting a new topic, I point out that even when they end up with the right answer, if they haven't used the method I taught then clearly the method they used was right. We then discuss what different methods you could use to come up with an answer (Appendix F.1). These interactions are clearly constructivist.

As students were engaged in exploring mathematical problem-solving from a constructivist perspective it became apparent that students worked well together

in their groups, and needed little guidance in relation to Polya's (1945) problem-solving procedure. Students were experienced in discussing and evaluating different methods in their attempts to solve mathematical problems. Students in Tomás' classroom engaged with each other productively, having productive discussions about the task at hand. Tomás explains that he prefers short periods of mathematical problem-solving because of their intensity. Students indicated in interview that they enjoy times when they work together. One student said: 'It was nice because you could work with your friends and if you couldn't get it, you could ask one of them' (Appendix F.2). Student interactions were conducted in an environment that welcomed discussion and debate. The following excerpt illustrates the extent to which students used trial and error, with little significant teacher interaction, in their attempts to solve the problem,

A farmer looks out into his barnyard and counts 14 animals – some horses and some chickens. He also counts a total of 40 legs among his animals. Can you figure out how many horses and how many chickens must have been in the barnyard?

Teacher: Remember all to pick out the important information and not to rush. Take your time and don't be afraid to try anything. Of you go in your groups now.

Student A: Well there are definitely 14 animals anyway because it says 14 heads.

Student B: Yes, and chickens and horses only have 1 head.

Student C: 32 feet, so a horse has 4 and a chicken has 2.

Student B: Pick a random number; so 7 horses and 7 chickens and that's it.

Student A: No, that wouldn't figure out; it is 28 and 14 which is too much so less horses.

Student B: 6 horses and 8 chickens – 6 horses is 24 and 8 chickens is 16. No, that is still too much.

Student A: Maybe 5 horses and 9 chickens

Student B: No

Student C: Is it am, 3 horses which is 12 and 11 so plus 22 which is 34?

Student B: 5 and 9 no, that wouldn't work either.

Student A: What is 2 horses, it is 8 and then …?

Student B: 12 chickens which is 24

Student C: That's it, 32 we got it.

Student A, B and C: It is 2 horses and 12 chickens.

Student A: Teacher, if 2 horses is 8 legs and 12 chickens is 24, so 32.

Teacher: Well done, how did you do it?

Student B: We just guessed it and found that the heads added to 14 and the legs to 32.

Tomás began the lesson be prompting the students to search the mathematical problem for the important information and reminding them to progress slowly. This introduction to the mathematical problem-solving lesson was common to all lessons conducted for the purposes of this research. The student interactions that follow clearly illustrate that Tomás' class are well able to identify and utilise particular strategies for solving mathematical problems without teacher direction. Their discussion led them to select trial and error as a method for solving the problem and all students contributed to this problem-solving episode. Students engaged in debate with one another, accepting one another's contributions and progressing quite quickly towards achieving an answer.

As well as reminding students of the important aspects of engaging with a mathematical problem Tomás also requested students to describe their behaviours on conclusion of the lesson. Tomás' interest in engaging students with mathematical problems in this manner stems from his own particular interest in and enjoyment of such activity during his own primary schooling, and also from his belief in the needs of the individual in society: 'Society needs people who can work around a problem and see things in a number of different ways' (Appendix F.1). He said: 'We spend too much time on computation, which is somewhat useful but really, of how much use is it compared with the ability to problem solve' (Appendix F.1). Tomás' students revealed that, although they do a lot of work on an individual basis such as 'multiplication and division' (Appendix F.2), they also 'do problem-solving and things like that in pairs' (Appendix F.2).

5.5.3 Tomás' emphasis on the rote memorisation of number facts

Tomás has clear conviction about the importance of the teaching of the basic facts of addition, subtraction, multiplication and division. He said: 'I drill them from the weakest to the strongest child. On a daily basis, we play table games so that they are well accustomed to their number facts. You can know your methods inside out but if your tables let you down you are at an extreme disadvantage' (Appendix F.1). Tomás explains that the difference between a 'good' mathematics student and a 'weak' mathematics student is the individual's capacity for problem-solving and that a 'weak' mathematics student can have a 'decent enough' capability in relation to computation because of the emphasis on mathematical number facts in the primary school. While engaged with the mathematical problems during the period of research, Tomás' students performed mental mathematics quite quickly and accurately. During interview, Tomás' students explained that they do 'lots of sums that take about a minute' (Appendix F.3).

5.5.4 Tomás' employment of group teaching methodology

As discussed, Tomás is comfortable with engaging students in co-operative learning situations, and students worked productively together on Tomás' chosen mathematical problems. Tomás arranged pupils in groups according to mixed levels of ability for the purposes of this research. The challenge, he revealed, was to keep all students in the group mathematically challenged. Tomás found he had particular difficulty with students with lower levels of mathematical ability. Tomás found that the students with good mathematical ability were capable of following problem-solving procedures discussed in class, but he found it difficult to ensure children of lower levels of mathematical ability were sufficiently engaged in the process. For this reason, Tomás explained that he would modify the format of the lessons so that children would be grouped by ability levels. This would, in his opinion, enable him to provide more teacher support to the students requiring it most. He explained, too, that the more able mathematics students would be capable of working quite well together because 'they can bounce off each other more' (Appendix F.1).

Students with lower levels of ability in relation to mathematics were taught essential problem-solving skills by Tomás. Tomás had to describe and model the four-stage problem solving procedure for these pupils. Tomás explained: 'It's not the problems that the teacher needs to be conscious of, rather the methodology or the teaching of it' (Appendix F.1).

5.5.5 Tomás' constructivist approach to teaching and learning

Tomás found the project to be a very worthwhile experience; he explained that he witnessed a 'significant improvement in students' problem-solving skills' (Appendix F.1) and would 'highly recommend approaching problem-solving in this manner' (Appendix F.1). Prior to engaging students with the mathematical problems Tomás spent time discussing Polya's (1945) four-stage problem solving procedure with the class. Tomás found he had to show by example and worked through some mathematical problems with students prior to engaging them in the actual experience. It is clear from the analysis of the constructivist episodes that Tomás was methodical in his explanation of an approach to mathematical problem-solving. Students were encouraged to engage fully with the problem and follow the procedures discussed. This is illustrated by the following:

Problem 1
Teacher: Remember all to pick out the important information and not to rush. Take your time and don't be afraid to try anything. Of you go in your groups now.

Problem 2
Teacher: Ok, where do you start, what is the important information now? What numbers do you need to focus on? What information is important and what information is unimportant? Use highlighters.
Problem 3
Teacher: This is a short problem but still, remember to check for information that may be important and unimportant and talk about anything that comes into your head about the problem with your group.

Tomás was consistent in the directions given to students. These directions were effective as students explained during interview, 'not everything is needed' (Appendix F.2), in reference to the information supplied by a problem. Students continued to reveal that after the initial lessons they realised that they had to 'talk about the important stuff and try different things if some didn't work' (Appendix F.2). Students were encouraged to offer advice to those who might not be familiar with how they solved problems throughout the period of the project and a selection of their responses proves that their lessons were very much constructivist in their design and execution.

- 'Take your time and be careful what you do.' (Appendix F.2)
- 'You don't need all of the information all of the time.'(Appendix F.2)
- 'Try working it out even if it might be hard.'(Appendix F.2)
- 'It's ok to get it wrong and ask for help.'(Appendix F.2)
- 'Think about things first before you do things; read it a few times.' (Appendix F.2).

5.6 Participant five: Mike

Mike was a participating teacher in a County Kildare school.

5.6.1 Introduction

Mike is an enthusiastic teacher with a broad range of experience as both a classroom teacher and a language support teacher. Mike teaches in a large urban boys' primary school. The children come form middle class backgrounds and resources are plentiful. Mike has a very high level of education having recently graduated with a Master of Arts degree in Language Education. Mike was very keen to engage with this project from the very beginning. In working with Mike it became very clear that he strives to ensure that all children are exposed to the key objectives of the mathematics curriculum. Mike is very aware of current trends and practices in relation to mathematics education and endeavours to try various methodologies and approaches to teaching mathematics in his classroom. Mike explains that any recommendations from curriculum support personnel are implemented in his classroom, and he enjoys engaging with innovative teaching methodologies. Mike feels, however, that there is a lack of cohesion amongst the partners involved in education, and that conflicting expectations can make the delivery of the curriculum difficult for the teacher.

It became evident as the project progressed that Mike had strong views regarding the exploration of mathematical problem-solving from a constructivist perspective and these will be discussed. However, Mike did find the experience valuable and he explained: 'It has opened my eyes; children do like engaging with problems and with themselves. They can learn a lot from each other, from teaching each other and by reasoning together' (Appendix G.1).

5.6.2 Mike's didactic approach to the teaching of mathematics

Mike explains that there is a positive correlation between a student's attitude to a subject and a teacher's style of teaching the subject. He said: 'If you like their

style and their teaching of maths, you will do well' (Appendix G.1). Mike found that learning to teach primary mathematics at third level was ineffective in preparing him for his profession. He explained that the lack of a 'concrete' approach to the explanation of mathematics teaching methods meant that he spent a significant amount of time researching the teaching of mathematics himself. While he acknowledges that the mathematics curriculum allows for an activity-based approach to learning he felt that it can be difficult to implement this in the classroom due to a variety of factors, including class size, the breadth of the curriculum, and the range of students' abilities.

Mike's approach to teaching mathematical problem-solving can be described as traditional. He highlighted, in particular, the teaching of the multiplication of fractions to children. He explained: 'You just have to sit down and say look, this is the rule for multiplying a decimal by a decimal and just do it' (Appendix G.1). It appears Mike does not normally use the type of problems chosen for exploring mathematics from a constructivist perspective on a regular basis; students in his class revealed that these problems were 'nice and more interesting' (Appendix G.2). They went on to say that the problems chosen for exploring mathematical problem-solving from a constructivist perspective 'were different and challenging unlike what we do' (Appendix G.2). Mike used mathematical problem-solving exercises to evaluate students' understanding of a concept taught didactically. He said: 'Throughout the term and the week, as you get to Friday, you try and get them to solve the problems based on material perhaps that you would have covered' (Appendix G.1). Mike defends his rigid use of mathematics textbooks, explaining 'students have to have a basis' (Appendix G.1).

Mike places significant emphasis on the operations: addition, subtraction, multiplication and division. He said: 'Children need to have a good understanding of the operations' (Appendix G.1). Students in Mike's classroom work independently of one another under the stewardship of the teacher. Mike reveals that engaging with a constructivist approach to the teaching of mathematics was difficult from his perspective because 'relinquishing control and that was difficult at the start … as teachers we feel we

have to be in control of the whole lesson, but we must have children test their own ideas and hypotheses '(Appendix G.1).

5.6.3 Mike's difficulty with a high pupil teacher ratio

Mike was particularly insistent throughout the period of research that using a constructivist approach to any subject area, and particularly in mathematics, is very difficult given the range of abilities in the average- sized primary classroom. Mike explains that it is the workload of the teacher that will determine the methodologies that are employed in his or her respective classroom. He said: 'If you have a large class and try to differentiate the activities, it is hard to make progress … people have to be practical and get real and acknowledge the problems in today's classrooms' (Appendix G.1). Mike finds it difficult to manage the individual needs of every child in a class containing thirty pupils. He explains that, to implement the Primary Mathematics Curriculum (1999) fully, teachers would have to plan for every child on an individual basis because of the broad range of abilities in the student population. For this reason, Mike explains that it is difficult to get past the point of that ensuring all students have a good understanding of the basic operations. Mike revealed that 'it moves quite slowly because of the range of abilities within the classroom' (Appendix G.1). He said that it might be appropriate to consider the role of the resource teacher and the part he/she can play in the classroom together with the class teacher. Mike also believes the streaming of students may also be an option.

5.6.4 Mike's constructivist approach to teaching and learning

Mike endeavours to integrate mathematics with other curricular areas, in particular, the Visual Arts. He explains that this is because he would describe himself as passionate about the creative arts. He says that other subject areas allow the teacher to reveal mathematics as a practical useful subject to students. He specifically highlighted lessons involving construction and an understanding of line and angle. Although Mike would not describe his approach to mathematics as particularly constructivist, his use of children's everyday

experiences, their environment, and involving them in the activities outlined above can be described as constructivist.

Mike can describe constructivism succinctly. He sees it as: '… a hands on approach to a topic, your starting point is the level of understanding of the children, it is about group work, getting them to come together and work co-operatively, testing out their own ideas and theories. It has children trying out ideas and revisiting them to make changes and alterations' (Appendix G.1). Mike explains that constructivism is not a primary methodology utilised by him in his classroom. He said: 'Bookwork takes over very quickly in first and second class … we need to take the lead from infant education … as you go up the school, constructivism is watered down a bit' (Appendix G.1).

5.6.5 Mike's use of mathematical language

Mike is quite convinced that children need to experience a wide range of mathematical language, particularly in problem-solving. Mike explained that, in many cases, when children are presented with a mathematical problem that uses mathematical language with which they are not familiar they cannot solve such problems, because of the restricted use of mathematical language within the classroom. He reveals that teachers need to differentiate and be careful to use a variety of mathematical language. Mike is mindful of the language he uses during mathematics lessons and this was evident as he engaged the children in the mathematical problem-solving episodes. Although his interactions with students during their mathematical problem-solving exercises could not be described as constructivist Mike was clearly emphasising the use of mathematical language. Mike's students also used mathematical language carefully and successfully.

5.6.6 Mike's understanding of a constructivist approach to the exploration of mathematical problems

Mike displayed a clear understanding of a constructivist approach to mathematical problem-solving during the semi-structured interview. During his

interactions with students, however, Mike was reluctant to allow them significant opportunities for engagement and discussion with one another. During the interview and in the course of professional development Mike displayed a good understanding of constructivism and its implications for the classroom, but explained that he had not employed to any significant extent in his planning for and teaching of mathematics. Mike described constructivist teaching as follows: '… a hands on approach to a topic, your starting point is the level of understanding of the children. It is about group work, getting them to come together and work cooperatively, testing out their own ideas and theories. It has children trying out ideas and revisiting them to make changes and alterations' (Appendix G.1). It is the variety of pressures that he experiences in the school that, in his opinion, restricts Mike in employing constructivist methodology. Interestingly, Mike described students' interactions with one another as constructivist, using phrases such as 'collaborating' (Appendix G.1), 'testing out hypotheses' (Appendix G.1), and coming up with their own strategies and modifying them if necessary' (Appendix G.1).

5.6.7 Mike's didactic approach to mathematical problem solving

However, in an analysis of the constructivist activities in which the children were involved, it appears that they were restricted in their development of hypotheses, in their experience of collaboration, and in the opportunities they were given to develop problem-solving strategies.

This is illustrated in the following examples.

Teacher: Okay Student Y, explain to me what you did so.
Student Y: I read over all of the instructions and then I read over 1, 2, 3 and 4. Then, the 2 is definitely in the thousands place. Then it says he remembers the number in the hundreds place is three times the number in the thousands place. I multiplied 2 by 3 which is 6, so 2 is in the thousands place. The number in the hundreds place is 4 times the number in the thousands place so I multiplied 4 by 1 which is 4 so it is 1462.
Teacher: That's incorrect you have one fundamental mistake.

Student Y: What!

Teacher: We will go over here to this boy.

Student Z: He said the number 2 is sitting in the thousands place so that was actually there so I put 2 down. He said he remembered seeing a number 1 so I just kept that in my head. He said in the hundreds place he remembered seeing a number three times the number in the thousands place. So I did 2 by 3 which is 6 so I wrote down six. Then he said the number in the ones place was 4 times the number in the tens place, so I just wrote down 4 there and I thought the 1 was just the units at the end so I did that by 4 and I wrote down 4.

Teacher: No that's not correct. Anyone else got it figured out?

Student Z: He said the number in the ones place is 4 times the number in the tens place. Ok, in the hundreds place he remembers the number is 3 times the number in the thousands place

Teacher: Okay, everyone listen now here. He said the number 2 is in the thousands place so 2 is in the thousands place. In the hundreds place he remembers seeing three times the number in the thousands place so that is 2 multiplied by 3 which is 6.

From the above, Mike provides significant guidance for students in their attempts to solve the problem. Mike does not utilise questioning in a facilitative manner; his questions are very direct and provide students with little opportunity to revise or examine difficulties in their solutions to the problem. Mike utilises phrases such as 'that's incorrect, you have got one fundamental mistake' (Appendix G.1) and 'no that's not correct' (Appendix G.1), and after asking all members of the group to explain their answer, and on still finding students had solved the problem incorrectly, decided to explain the answer to the problem himself. The lesson continued in this vein and concluded abruptly as evidenced below.

Teacher: Where does that go?

Student Z: Under the hundreds. He said the number in the ones is 1 so we put the 1 there and the 4 in the ten.

Teacher: Why did you put the 4 there?

Student Z: Because you told me to, I don't know?

Teacher: Look at number 3. He said the number in the 1's place is 4 times the number in the tens place so the answer is 2614 not 2641. Do you understand where that came from? You had yours backwards. Some people started with the units instead.

There was no evidence of engaging students in Polya's (1945) four-stage framework of problem-solving. As evidenced from the conclusion to the lesson, students did not have time to reflect on their answers and also, during the lesson, children were given little time to respond to the teacher's interactions.

The following excerpt from a problem lesson reveals similar interactions with pupils. Mike provides direct assistance to students as they labour to find a solution to the problem.

Teacher: Firstly, read the question together.

Student A: Farmer Tom put a square fence around his vegetable garden to keep deer from eating his corn. One side is 10m in length. If posts were placed 2m apart, how many posts did he use?

Student B: So length is 10 here, so it is the same on the bottom as on the sides.

Teacher: Why have ye labelled the sides 2m, shouldn't it be 10m?

Student A: Should we do area and all that it might be right to do that?

Student B: You know the way it says posts; where are the posts, we don't know. They must be at the side or something.

Teacher: Look it says the posts are going to be 2m apart around the edge.

The teacher draws and shows them.

On this occasion Mike draws a sample of the solution for the pupils and explains to the students the errors they have made in their initial problem-solving efforts. He said: 'Why have ye labelled the sides 2 metres, shouldn't it be 10 metres' and 'Look it says the posts are going to be 2 metres apart around the edge'. After this the teacher draws an example to illustrate the point.

Both the above interactions and the structure of the lessons organised by Mike are not constructivist and they do not follow procedures examined during professional development. There is a clear disparity between the actions of the

teacher in the classroom and the beliefs held by the teacher. Interestingly, Mike explained during interview: 'I suppose perhaps I was taking a step backwards, relinquishing control and that was difficult at the start. As teachers we feel we have to be in control of the whole lesson, but we must have children test their own ideas and hypotheses, we have to obviously take a look at the role of the teacher in all this' (Appendix G.1). During final discussion Mike revealed: 'I would have felt at times, what am I achieving here overall? At times I said to myself, I have a maths programme that does not appear to be covered in this. What is the end objective of all this, the visible results? They are not doing traditional bookwork that is expected of me by all the partners here. I think it is ingrained in me that there must be quantifiable results visible regularly' (Appendix G.1).

This became a recurring concern for Mike throughout the project. He was worried for his students and what they were achieving. This may explain his significant interactions with them during problem-solving. Also, Mike mentions the partners in education, including the board of management, the parents, and the students. Mike feels there must be visible results for these partners to witness and feels that constructivist teaching practices will not provide such results.

5.6.8 Mike's use of group teaching methodology

Mike devised mixed ability groups in his classroom for the purposes of this research. He felt challenged in that he explained that the students with good levels of mathematical ability were dominating the exercises and those with lower levels of ability were, in Mike's words, 'lost' (Appendix G.1). For this reason, Mike explained that he would modify the format of the lessons so that children would be grouped by ability levels. Mike warned that teachers would need to have a 'sound evaluation of their understanding before setting tasks and designing groups' (Appendix G.1). Students in Mike's classroom indicated their interest and enjoyment in working in collaborative group situations: 'It was like normal problem-solving but just with your friends' (Appendix G.2). Similar responses indicated that group collaboration in Mike's classroom was a

novel experience: 'It's easier when your friends help you out sometimes instead of the teacher' (Appendix G.2). 'The only difference was that we were in a group and not on our own' (Appendix G.2).

5.7 Conclusion

Chapter Five has revisited the cases of Susan, Emily, Joe, Tomás and Mike and has organised the data gathered throughout the period of the research on a case by case basis. It became apparent that a number of common themes emerged across all of the five cases and that the implications for constructivist methodology, as outlined in the Primary School Curriculum, have far reaching consequences for teachers given their experience and knowledge to date. Factors that impact on the exploration of mathematical problem-solving from a constructivist perspective have been revealed and discussed and will be reflected upon in chapter six which will draw together all of the cases with reference to relevant literature.

Chapter 6

Discussion

'What reformers might see as trivial … teachers would estimate as grand revolution, especially as they were just beginning to change' (Cohen, 1990:325)

6.1 Introduction

The previous chapters presented portraits of five participant teachers comprising a detailed picture of their exploration of mathematics in their classrooms, their opinions on effective mathematics teaching practices, their actions as they engaged with mathematical problem-solving from a constructivist perspective, and their thoughts on implementing constructivist practices in the primary mathematics classroom. The cases of these teachers have been presented individually to capture their discrete significance. Following the examination and analysis of these case studies, this chapter will now discuss them to determine implications for future practice. Despite variations in age, education and experience, the findings suggest common strands of thought and attitude among the participants. Strong opinion emerged across all of the cases in relation to what has been termed traditional school mathematics (Schoenfeld, 2004) and, in particular, the employment of a drill and practice approach to teaching basic mathematical facts and algorithms. Similarly, strong opinion emerged across the cases in relation to teaching large numbers of students from a constructivist perspective, engaging with students as a facilitator of the student's learning, utilising group work as a method of instruction, managing the efforts of students with different learning abilities, and utilising constructivist experiences as enrichment activity. This chapter will focus on participants' opinions in relation to the above but also discuss generally their constructivist approach to mathematical-problem solving by reflecting on their

mathematical problem-solving lessons and their thoughts and reflections about engaging with constructivist methodology.

6.2 Traditional versus reform mathematics

Cohen (1990) describes the case of Mrs. Oublier, one teacher's response to reform mathematics in the state of California during the 1980's. Mrs Oublier found a new way to teach mathematics having spent her initial years teaching mathematics through the memorisation of facts and procedures. Some observers would agree a revolution had occurred in Mrs. Oublier's classroom but others saw only traditional instruction. Cohen (1990: 312) concluded: 'Mrs. O. is both of these teachers. As teachers and students try to find their way from familiar practices to new ones, they cobble new ideas onto familiar practices' (Cohen, 1990: 312). From the researcher's engagement with Irish primary mathematics teachers, this blend of the traditional and reformed was quite evident across the five cases. Classrooms are, by their nature, regimented, controlled environments and the classrooms of the participating teachers were no different. All of them were typical of the Irish primary situation. As generalist teachers they explored a broad and varied curriculum with large groups of children with wide spectrums of ability in relatively confined spaces. All of these factors influenced the teachers significantly as they engaged with teaching mathematics from a constructivist perspective. However, in engaging with this project, every participant indicated a keen interest in mathematical problem-solving and constructivist practices and acknowledged the significance of mathematics, and, in particular, problem solving. The enthusiasm shown for constructivism was tested as teachers endeavoured to put policy into practice. Wolfe and McMullen (1996) have explained that although constructivism may influence teaching, it is a theory of learning with implications for teaching so therefore interpreting these implications can be difficult. Airsian and Walsh (2007) explain that constructivism is a theoretical framework that offers teachers little detail in the art of teaching.

Pirie and Kieran (1992) explain that the creation of a constructivist classroom is a significant task for a teacher as it involves much more than textbook chapters

and seat work. Constructivism is a theory of learning that must be translated by the teacher into a theory of teaching. There are also other significant factors that were highlighted by this study that impacted on the successful transition to using constructivism as a basis for teaching in classrooms. These factors were the large numbers of students that had to be taught, the breadth of the curriculum, the wide range of abilities that an individual teacher has to contend with, and the challenge of managing student learning in group situations. In the context of practicing teachers' beliefs in what constitutes appropriate mathematics teaching, and teaching practices that are employed on a daily basis, the creation of a constructivist classroom was a significant task for participating teachers. These teachers displayed a keen respect for traditional mathematics instruction, including the need for significant focus on computation and the importance of students recalling basic mathematical facts acquired through rote memorisation. At the end of their engagement with the research project it was clear that such methods of instruction and focus on content will continue to be utilised frequently and that although teachers were inspired by constructivist methodology, it will not be a primary methodology used in their classrooms.

Geelan (1997) explained that when teachers first encounter constructivism, it can appear as a simple, but superior, epistemology that has implications for teaching. This study highlights the implications of engaging with the constructivist learning perspective and offers insight into structuring effective lessons from an emergent perspective. As the participant teachers engaged with constructivist practices in relation to mathematical problem-solving during their professional development, it became clear that four of the participants found it 'refreshing' (Appendix D.1), indicating that it would be a significant deviation from the norm to employ this methodology in the classroom. All five participants found discussing and examining constructivism and its implications for their particular situations exciting and were keen to engage with it in practice. Given that the basic principles of the Primary Mathematics Curriculum (Government of Ireland, 1999a; 1999b) are constructivist, this was indeed surprising; it should not have been revolutionary to them. Indeed, all of the teachers had experienced in-service education provided by the Department of Education and Science that dealt specifically with the Mathematics curriculum.

Enthusiasm for engaging with constructivism during professional development cannot be understated and, although participants appeared to have limited understanding of such approaches to mathematics, they were eager to see them in practice in their own classrooms. Constructivism may be a primary principle of the Primary Mathematics Curriculum (Government of Ireland, 1999a; 1999b), but it has yet to make the transition from the curriculum to the classroom. Importantly, the NCCA (2008) in their review of the implementation of the primary curriculum stated that teachers have difficulties with engaging students in collaborative learning.

The teachers had significant reservations about the employment of constructivism in the mathematics classroom once they engaged their students in mathematical problem-solving from a constructivist perspective. Cobb, Wood and Yackal (1988) explain that this is because teaching from a constructivist perspective breaks the mould radically from traditional educational models in which teachers themselves were schooled. Susan's deeply held and strong beliefs about teaching, classroom management and curriculum were factors that influenced her involvement with constructivism when it was taken from the professional development initiative to the classroom. During her mathematical lessons these beliefs and attitudes became evident in her view of constructivism as primarily an enrichment activity for capable students of mathematics, in the importance she placed on of rote memorisation and the practice of traditional school mathematics, and her perspective on the role of the teacher in student deliberations. Rote learning was utilised by all of the participating teachers, and it is interesting to note that Von Glasersfeld (1990) and Noddings (1984) explained that rote learning has no place in constructively oriented instruction and suggest that, from a constructivist perspective, rote learning limits the students' ability to think more deeply. Given this however, when time is a constraining factor, rote learning is often regarded as a more efficient way of teaching with young students, (von Glasersfeld, 1990). Susan felt that, because of the varying levels of ability in the classroom, the content to be covered, and the lack of basic mathematical knowledge of the part of these students, utilising a constructivist approach to mathematics would not be of significant benefit them. Mike also explained that

other pressures, such as those stemming from the school management and the Department of Education and Science, will determine the instructional practices that will be employed in the classroom. Emily explained that she teaches the number strand thoroughly to students using traditional approaches and would only consider engaging with a constructivist approach when she feels confident her students have significant background knowledge. This is a valid point. Constructivism offers very little detail in the art of teaching and therefore traditional methods of instruction particularly in mathematics continue to play strong roles in classrooms. Traditional methods of instruction allow teachers explore content in detail and in a specific timeframe. The role of the teacher from a constructivist perspective is not adequately addressed and needs elaboration.

Traditional approaches to instruction in mathematical problem-solving amongst participating teachers were evident, particularly in the interactions between teachers and students during mathematical problem solving. From the data that has been presented, teachers found it difficult to redefine their relationships with students during instruction in mathematical problem-solving when moving from a didactic to a more facilitative role. It restricted student learning from an emergent constructivist perspective. This was particularly obvious in the cases of Susan, Joe and Mike. From an examination of their problem-solving lessons, students had very little opportunity to fully experience a constructivist approach to learning as the teachers involved endeavoured to have a traditional relationship with students as they solved problems. This was particularly evident during students' engagement with stage four of Polya's (1945) problem solving heuristic. Current empirical research specifically reveals the importance of having students justify solutions and fully exploring extension activities that may emanate from the problem solving exercise with them (Elmore, 1996; Francisco and Maher, 2005; Hoffman and Spatariu, 2007). There was limited engagement with students in stage four of problem solving outside of recapping on work completed during the process.

Wilburne (2006) explains that a problem solving heuristic should be used to foster mathematical thinking and develop mathematical students' ability to

solve mathematical problems. The stages espoused by Dewey (1933) and modified by Polya (1945), to make it more specific to mathematics, are typical of a socioconstructivist learning environment in which ideas and strategies are shared with significant levels of experimentation and interaction. Throughout the professional development initiative, as teachers came to an agreement about how problem solving might best be explored from a constructivist perspective, strong consensus emerged amongst teachers regarding the use of Polya's (1945) heuristic. As an initial exploration point for teachers beginning to translate a theory of learning such as constructivism into a theory of teaching, Polya's four stage problem solving procedure is invaluable. However, the heuristic is only as effective in the manner it is employed. Teachers' use of Polya's heuristic varied but what was common across all the cases was the interpretation of the fourth and final stage, the stage of reflection. Teachers utilised this stage to discuss students' solutions to mathematical problems and the depth of these discussions varied as illustrated in previous chapters. However, common across all cases was the failure of participants to generalise from or extend the various solutions presented. It is during this stage and from such generalisations and extensions that students can build and design more efficient problem solving strategies for use in future situations. Key activities associated with this stage such as forming predictions, making interpretations and engaging in debate (Windschitl, 1999; Francisco and Maher, 2005) were not in evidence throughout this study. Furthermore, Greer (1997) explains that by engaging in the activities listed above students become flexible in the comprehension of future problems. Teachers need to focus on student deliberations and solutions and identify how the strides made in the solving of mathematical problems on specific occasions by students can be extended and used elsewhere.

Teachers used Polya's format (1945) but conducted the exercises in a thoroughly traditional fashion. Teachers were helping children to arrive at answers and it is clear from these episodes that, although the teacher's involvement with students resulted in the achievement of answers, student understanding of the answers and the techniques used to get them were limited. This was particularly evident in Joe's case. Cuban (1984) explained that many teachers construct hybrids of particular progressive practices grafted on to what

they ordinarily do in classrooms (Cohen, 1990; Cuban, 1984). The case of Joe was particularly interesting, even though it was clear he believed that the employment of mathematical problem-solving and the development of independent problem solvers should be the primary goals of mathematics lessons, his teaching of mathematical problem solving was very evidently didactic, and thus prevented the pupils from developing this autonomy. His interactions, particularly his questioning strategies, with pupils and his provision of direction as they solved problems bore little resemblance to a constructivist approach. Joe's interactions with students were quite different from what Gallimore and Tharp (1989) and Draper (2002) explain are more appropriate to organising learning for students from a constructivist perspective. According to Gallimore and Tharp (1989) and Draper (2002), students must be enabled to read, write, speak, compute, reason and manipulate both verbal and visual mathematical symbols and concepts. To achieve this, the teacher should use an elaborate set of strategies including questioning, inferring, designing, predicting and facilitating.

Teachers' own school experiences significantly influence their approach to teaching (Lortie, 1975). This became particularly evident in those that recalled enjoyable and, in their view, appropriate teaching practices. Both Susan and Tomás recalled their mathematical experiences at primary level and both experienced very different forms of instruction. Susan, who was taught in classrooms employing very traditional methodologies, holds such practices in high esteem and continues to identify with and utilise such approaches in her classroom. These practices include having children learn basic mathematical facts through rote memorisation and paying particular attention to the teaching of the operations. Susan did, however, incorporate elements of a constructivist approach to mathematical problem-solving into her didactic approach to mathematics following engagement with professional development, but her fervently held beliefs about what constitutes appropriate mathematics teaching were not changed in any significant way. Tomás' only significant recollection of mathematics at primary level was one that involved mathematical problem-solving at fourth class level. Tomás, having experienced success in problem-solving at primary level, identified with having children approach mathematical

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problems from a constructivist perspective, and elements of his teaching of mathematics were clearly constructivist even prior to engaging with professional development. Even those participants, whose mathematical experiences, in their opinion, were not in anyway significant found value in methods of instruction of former teachers. Hence, these teachers clearly implemented these methods in some format in their own mathematics classrooms. Interestingly, Emily, who experienced corporal punishment at primary level during mathematics class when she failed to answer mental mathematics questions, uses the same procedure for examining students' mental mathematics in her own classroom today. Emily lines up her students and asks them a series of questions very quickly and if these questions are not answered the student is asked to sit down, and so it continues until one member of the class is left standing.

6.3 The Irish primary mathematics classroom

It is not easy to approach the teaching of mathematics from a constructivist perspective given the factors at play in the Irish primary classroom. Although the participating teachers eagerly embraced constructivism at the outset, the realities of their classrooms, when they returned to implement constructivist methodologies, began to inhibit them. All the teachers taught large classes of over thirty students in school buildings designed and built during the 1970's. Space within these classrooms was at a premium and it was evident, as teachers engaged their students in mathematics from a constructivist perspective, that more traditional methods of teaching are far easier to employ. Tomás explained that the intensity of pupil interactions with problems from a constructivist perspective led him to being capable of managing only short sessions regularly. Windschitl (1999) explains that mathematical problem solving constructivist classrooms are highly charged environments involving considerable debate, discussion and argumentation. Tomás explained that the level of co-ordination and management required by constructivist lessons can be demanding on the teacher. He found that student interactions can be loud and disruptive, and in the classrooms of today, this can be difficult to tolerate on a sustained basis. It takes little effort to use traditional forms of instruction in mathematics, and

these forms of instruction provide identifiable gains in the short term. Susan revealed that delivering the content of the curriculum and covering all of the strands and strand units requires teachers to move swiftly through their programmes of work. This, together with the various learning styles of the pupils in her particular classroom, was a significant factor in her conclusion that utilising a constructivist approach to the teaching of mathematical problem-solving was suited only to those pupils requiring enrichment activities.

Two of the participating teachers had strong views about this particular issue. Both Mike and Susan explained that, given the nature of constructivism, it is unreasonable to expect that in small classrooms with large numbers of pupils the teacher could develop teaching episodes designed to attend to every particular child's level of understanding and learning. Key features of classrooms that foster learning from an emergent perspective involve students working collaboratively with appropriate tools and resources on diverse mathematical problems (Windschitl, 1999). It can be difficult to coordinate this type of work in classrooms with large numbers of students. Furthermore, Mike explained that the pressures from various partners in education tended to direct teachers towards covering a wide curriculum in a short time frame and, therefore, did not permit significant engagements with mathematical problem-solving from a constructivist perspective. Airsian & Walsh (1997) reveal that for students to make strong connections, teachers must listen, respond and structure further learning opportunities which can be time consuming. This is a significant problem arising from using constructivism as a referent for teaching and learning.

6.4 Teaching mathematical problem-solving

Wilburne (2006) has explained that the best mathematical problems one can employ in the classroom are non-routine mathematical problems that encourage rich and meaningful mathematical discussions, those that don't exhibit any obvious solutions, and those that require the student to use various different strategies to solve them. These problems are best explored from a constructivist perspective but it becomes clear, from analysing the cases of the teachers

involved, that mathematical problem-solving is envisaged quite differently by teachers.

A constructivist approach to engaging students with problem-solving was acclaimed by all participants as being effective in the development of students' capacity for problem-solving. However, although mathematical problem-solving occurred in the primary classrooms of all of the participants prior to this research, it is worrying to examine the role that it plays. The nature of children's engagement in mathematical problem-solving was restricted. One of the participating teachers, Mike, explained: 'Throughout the term and the week, as you get to Friday, you try and get them to solve problems based on material perhaps that you would have covered' (Appendix G.1). Utilising mathematical problem-solving in this manner is common in the Irish primary mathematics classroom (O'Shea, 2003), and implies that the teaching of mathematical problem-solving is not particularly explicit. In fact, as Tomás came to the end of the period of research, he revealed he became acutely aware of the need to actually teach problem-solving, which had not occurred to him previously. The employment of very traditional methods of teaching as revealed by Susan, Emily, Joe and Mike imply that students do not experience mathematical problem-solving in its own right. The participating teachers understanding of constructivist philosophy prior to engaging in this research suggested a limited understanding of a methodology designed to enable students to become independent, able problem-solvers. The traditional methods of instruction coupled with the strong focus on school mathematics, evident in all cases, prevent primary students from experiencing and experimenting with mathematics. This, together with a wide curriculum that has to be covered, impacts significantly on a teacher's ability to engage children in productive problem-solving. Francisco and Maher (2005) explain that students must be provided with the opportunity to work on complex tasks as opposed to simple tasks as such tasks are crucial for the development of mathematical reasoning.

Tomás, who identifies with a problem-solving approach to teaching and learning and espouses methods of teaching mathematics that are considered reformed, explained after engaging with the project that it was only when he

saw the enjoyment and learning opportunities that accrued to children as they interacted and co-operated with one another did he realise the importance of problem-solving as an entity in itself for students. One must bear in mind that Tomás is a teacher who recognised the importance of actual problem solving at the outset, over and above traditional mathematics.

6.5 Teaching mathematical problem-solving to students with different learning abilities

All participating teachers had concerns for those pupils who, they explained, had particular difficulties with mathematics and mathematical problem-solving. All participants explained that mixed ability group situations were less than satisfactory for the student who might struggle with the subject. Yet, Palinscar, Brown & Campione (1993) explain that teachers should arrange learning exercises in which students are encouraged to assist each other. The less competent members of the team are likely to benefit from the instruction they receive from their more skilful peers who benefit by playing the role of the teacher. With the exception of Joe who explained that 'the presence of a pupil who has flair for the subject can be a big help, he can bring the rest of the students with him' (Appendix E.1) all others revealed particular concern and indicated adaptations they would make in the grouping arrangements. Tomás revealed that significant scaffolding is required for the students with particular learning needs in relation to mathematics, and indicated that, by grouping students according to ability and structuring the amount of teacher interaction required by the various groups within the classroom, the constructivist explorations would be more beneficial to low achieving pupils.

It appears that mathematical problem-solving is experienced essentially by those pupils who have mastered basic computation and number work. Mathematical problem-solving, from the teacher's perspective, is viewed as an enrichment activity. In Susan's situation, her particular class have been viewed by past teachers and indeed by the school generally as especially low achieving and, as Susan revealed, have not, therefore, spent significant time problem-solving, but rather have been in classrooms for their entire school going lives where they

have experienced traditional methods of instruction in traditional school mathematics. Indeed, Susan explained that a constructivist approach to the teaching of mathematical problem-solving would be more suitably employed with a class of brighter pupils, which in her case would happen in the next academic year.

6.6 Development of classroom mathematical traditions

Cobb, Wood, Yackel & McNeal (1992) argue that students and the teacher must be actively involved in the development of their own classroom mathematical traditions. Cobb et al. (1992) reveal that classrooms should involve creative thinking, collaborative approaches, and teachers who are facilitators of learning. Facilitators of learning use conflict resolution and mutual perspectives to move students towards socially negotiated accepted meanings. These were key practices utilised by participating teachers during mathematical problem-solving from a constructivist perspective. Students who solved mathematical problems in classrooms that adopted constructivist practices were comfortable in challenging each other, and they engaged in appropriate debate and discussion as evidenced in their mathematical problem-solving sessions. Greer (1997) explains that when students take it upon themselves to question each others ideas and assumptions, it helps them to become flexible in the comprehension of future problems. Shared understanding was encouraged and supported by the teacher. The primary instructional routine of these teachers involved questioning and having students explain to each other the details of mathematical relationships that they uncovered. Students were not passive recipients of knowledge but engaged in the construction of mathematical knowledge by interacting with the teacher and each other. Emily's case is particularly interesting. She can be described as a traditional teacher, but it is very interesting to note that as she became involved in the research both she and the students became altogether engrossed in mathematical problem-solving. Both students and teachers displayed a zest for engaging with one another in non- routine mathematical problems. It became customary in their mathematical classes to engage with both mathematical problems and, as Emily revealed, Sudoku puzzles on a regular basis. This deliberation between student

and teacher in the development of classroom mathematical traditions led to the establishment of an exciting, highly energised mathematical classroom enjoyed by both student and teacher.

6.7 Conclusion

Airsian & Walsh (1997) cautioned that constructivism can be seductive and considerably more challenging than might be anticipated. It is a theoretical framework, which broadly explains the human activity of knowing but offers teachers very little detail in the art of teaching. Constructivists foster interactions between students' existing knowledge and new experiences, which is radically different from the traditional transmission model. Constructivist theory puts the onus on the students to construct their personal meanings and interpretations in order to achieve understanding. Teachers face many dilemmas in the adoption of a constructivist approach to teaching, not least in finding a balance between individual and group learning and the definition of appropriate constructivist instruction. These dilemmas were prevalent as teachers grappled with constructivism and its implications for the mathematics classroom from the inception to the conclusion of the project, and are difficult to resolve given the nature of the Irish primary classroom and societal expectations. The most profound challenges for teachers are: to make personal sense of constructivism, to re-orientate the culture of the classroom to accommodate constructivist philosophy, and to deal with conservatism that works against teaching for understanding (Purple & Shapiro, 1995). Teaching from a constructivist perspective proved engaging for students and teachers when both the roles and responsibilities of everybody in the classroom were explicit. Students were successful in their mathematical problem-solving interactions when they were presented with mathematical problems that reflected their current level of understanding yet were challenging, and when they followed the four-stage procedure set out in advance for problem-solving. Teachers who chose appropriate mathematical problems of relevance and interest to students and their capabilities, and who adopted facilitative roles employing open ended higher level questioning techniques, laid solid foundations for effective constructivist explorations. There is a fine balance to

be struck in achieving this as different individuals' social and cultural contexts differ; people's understandings and meanings will, therefore, be different (Airsian & Walsh, 1997). Students' construction of both strategies and solutions surprised individual students and teachers and initiated rich debate and interesting strategies of solution in those classrooms.

If teachers accept constructivism as a teaching approach, they must decide on how much emphasis can be placed on viable and meaningful constructions. Students will construct many feasible mathematical ideas and connections, and the role of the teacher will then be to challenge students to justify and refine these. There are significant issues that arose throughout the course of this research that should be addressed before implementing constructivist practices in the classroom. Teachers and students need to agree on what constitutes viable mathematics. Throughout this research it emerged that teachers' beliefs and attitudes regarding appropriate mathematical constructions did not correlate with what students may have deemed appropriate. In guiding students in their mathematical learning, but also in the development of intellectual autonomy, all partners need to work out what constitutes a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution (Yackel & Cobb, 1996), and also an acceptable mathematics solution (Cobb, Wood, Yackel, & McNeal, 1992). In Emily's case, students constructed mathematical solutions that Emily herself acknowledged she would not have thought of; however, these solutions were accepted because of the detail of the student explanations and written work. Teachers need to guide and evaluate students' learning within the classroom yet must be mindful that the provision of such guidance and their attempts at assessment must not undermine the development of a student's mathematical autonomy.

The Primary Mathematics Curriculum (1999) incorporates constructivist principles and recommends constructivist approaches. Teachers are the mediators of this curriculum and this study has highlighted specific issues with the engagement of pupils in constructivist practices. The review by the NCCA (2008a) of the implementation of the Primary School Curriculum has also revealed difficulties teachers have with engaging classrooms of students in group collaborations, which are central to constructivist methodology. The

primary difficulty with employing constructivist methodology is that it is a fundamental shift from what is conceived as traditional classroom practices. Strongly held convictions cannot be changed unless the people who teach and learn want to change, take an active part in changing, and have the resources to change (Cohen, 1990).

Chapter 7

Conclusions and Recommendations

7.1 Summary

The key focus of this research was to investigate the teaching of mathematical problem-solving from a constructivist perspective in Irish primary classrooms following the engagement of primary teachers in a professional development initiative involving constructivism. In attempting this, the researcher co-ordinated and managed a series of sessions on constructivism and mathematical problem-solving and then investigated participating teachers' exploration of mathematical problem-solving from an emergent constructivist perspective in their classrooms while examining their students' mathematical problem-solving explorations in a constructivist environment.

7.2 Introduction

It emerged from this research that participant teachers had particular difficulty sustaining a constructivist approach to mathematical problem-solving following their engagement in professional development involving constructivism. Wolfe and McMullen (1996) explain that although it may influence teaching, constructivism is a theory of learning and not a theory of teaching. Therefore, it can be difficult to translate this theory of learning into a theory of teaching. There were several factors that impacted on teacher's attempts to re-orientate their classrooms to reflect constructivist principles consistent with current research (Airsian and Walsh, 1997; Windschitl, 1999). However, this research endeavours to offer suggestions for enabling this translation to happen. Participating teachers had various levels of experience in the classroom in teaching primary mathematics, but the outcomes of their engagement with this

research are very similar. Participating teachers showed a keen interest in the principles of constructivism and the teaching of mathematical problem-solving from a constructivist perspective. They conducted mathematical problem-solving lessons from a constructivist perspective successfully, but the realities of classroom teaching, curriculum content, and ingrained beliefs in what constitutes appropriate mathematics teaching often affected their engagement and, consequently, their trust in a constructivist approach to the teaching of mathematical problem-solving. Pirie and Kieran (1992) concluded that teachers have distorted the original notion of constructivism because they wanted to be perceived as doing the right thing.

7.2.1 A framework for a mathematical problem-solving lesson based on constructivist teaching methodology

In engaging with participating teachers prior to entering their classrooms it was clear that every teacher brought a different perspective about mathematical problem-solving to the research. A common thread in participating teachers' thoughts and reflections was that mathematical problem-solving is used as a means to an end. This is problem solving as context (Stanic and Kilpatrick, 1989). Schoenfeld (1992) explains that this is viewing problem solving as a means of facilitating the achievement of other goals. In the situations investigated during the course of this research, the engagement of students in mathematical problem solving came second to an exploration of the strands of the mathematics curriculum using traditional methods of instruction. Mathematical problem solving as art (Stanic and Kilpatrick, 1989; Schoenfeld, 1992) was not a characteristic of the classrooms of participants and one of the successes of this study was the realisation by participants of the importance of mathematical problem-solving itself. Tomás, who can be described as having an understanding of the importance of mathematical problem-solving revealed that it is only when you see children actively engaged with a mathematical problem that you realise the importance of having children solving problems for problem-solving purposes only. This was one of the important successes of this study.

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Participants agreed that utilising Polya's (1945) framework for mathematical problem-solving was useful in getting students to engage in positive interactions with one another about mathematical problems. Recent research concluded that using such a heuristic enables the teacher to foster mathematical thinking and develop students' ability to solve mathematical problems (Wilburne, 2006). Students involved in this research had varying degrees of experience in solving mathematical problems in co-operative group situations and, by using Polya's framework, made the transition effortlessly from working on an individual basis to working as part of a group. During the initial stages of the research students found collaborating as part of a team strange, particularly in Joe's case, but having a framework that was discussed in advance with students made it easier for the teacher to facilitate students in their interactions with one another. It also made the management of groupwork situations easier for teachers as there were clear guidelines included with each of the four stages. Students enjoyed developing their own ideas and recalling particular problem-solving strategies that they had utilised in previous explorations, and using Polya's framework helped the teacher structure his teaching to facilitate this. Using this framework also enabled teachers to renegotiate their role in the constructivist classroom. Throughout the cases it can be seen that implementing a particular procedure such as Polya's (1945) allows the teacher to take a step back and assume a more facilitative role, because students are clear about what is expected of them during the lessons. The nature of the teachers' involvement with the students as they solved problems varied greatly as all of the cases illustrate, but the employment of Polya's (1945) framework ensured that children had a clear understanding of the process of problem-solving.

As students solved particular problems, engaging them in the final stage of Polya's (1945) problem-solving framework, 'look back', allowed the teacher to understand the particular problem solving strategies chosen by the students to solve the problem. Although the full potential of this stage was not realised across all of the cases, this stage of the framework was crucial as, in the case of Emily, it surprised the teacher and revealed to her what the children were able to do, often far more than what the teacher might have given them credit for. This stage allowed teachers to ascertain the type of problem that would suit the

particular children in their class. However, this study proved that having students debate, generalise and extend methods of solutions, which are key activities related to stage four, is difficult to facilitate in classrooms that intend to structure learning from an emergent constructivist perspective. This final stage is the most crucial stage in ensuring that what has been learned can be transferred to future problem situations (Schoenfeld, 1992).

7.2.2 The impact of constructivist teaching methods on mathematical problem solving explorations

Mathematical problem-solving played a variety of roles in the classrooms of participants as is illustrated in the presentation of data. Schoenfeld (1992) distinguishes three traditionally different views of problem solving. In one, problem solving is an act of solving problems as a means to facilitate the achievement of other goals such as teaching math. In another, problem solving is a goal in itself of the instructional process. It is a skill worth teaching in its own right. Finally, problem solving involving challenging problems can be viewed as a form of art as what math is ultimately about. Utilising constructivist teaching methods made problem-solving more explicit in the classrooms and students were absorbed in the problem-solving process. As illustrated in the cases of Susan, Mike and Emily, problem-solving had played a supportive role to more traditional teaching methods. Problem-solving became a topic in itself being explored, and the use of constructivist teaching methodology made this a success. Teachers began to appreciate the need for students to engage with the mathematical problem, the selection of appropriate problem-solving strategies and with each other in negotiating a solution to the problem. This enabled teachers to see the value of mathematical problem-solving from a constructivist perspective. All participants, at the conclusion of the period of research, acknowledged the intrinsic value in approaching mathematical problem solving from a constructivist perspective. Pupils adapted well to what was, in some of the cases, a radical shift in teaching methodology and, also, exhibited capabilities that teachers may not have felt they possessed. Teachers began to recognise the power of children's constructions as they solved problems by exploring patterns, making conjectures, justifying solutions

and testing hypotheses (Francisco and Maker, 2005; Hoffman and Spatariu, 2007).

7.2.3 Participating teachers' exploration of mathematical problem solving from a constructivist perspective

Participating teachers exhibited eagerness at the outset of this research. Although they differed in terms of age, background and experience, all held mathematics and the teaching of mathematics in great esteem and were particularly eager to become involved in the teaching of problem solving. It has been explained that problem-solving played a variety of roles in the classrooms of participants. Indeed, as in the case of Mrs. Oublier (Cohen, 1990), as teachers began to teach from a constructivist perspective it was evident that different teachers had different interpretations of what it meant to use constructivism as a methodology for teaching. There is a gap in literature surrounding the emergent perspective that will be discussed later. Also, due to the intrinsic difference between constructivist methodology and traditional classroom methods, teachers adopted different approaches to the mathematical problem-solving lessons. Some of the participants had no difficulty in adopting a constructivist approach to teaching while others employed methodologies that could be described as a mixture of traditional teaching practices and constructivist methodology. Constructivism will have different meanings for different individuals; only sustained efforts in professional development directed at supporting the teacher in the primary classroom will help make the shift from traditional methods of teaching mathematics to constructivism, if that is what we require. Teachers, at the outset, displayed a limited understanding of constructivist methodology; it was a novel methodology that they agreed to engage with in their classrooms. Although the mathematics curriculum espouses constructivist principles, and the in-service delivered to teachers reflected those principles, the traditional understanding of and approach to mathematics teaching that was characteristic of the teachers inhibited the acceptance of a constructivist approach.

7.2.4 Student explorations of mathematical problem solving in a constructivist environment

Designing a constructivist environment and engaging students in constructivist collaboration was quite a task for teachers given their varied experiences. Constructivist environments are lively and full of energy (Windschitl, 1999) and this can be difficult to become accustomed to, as participating teachers alluded to in this study. The demands teachers experienced in employing constructivist teaching methodologies are great, not least because of the numbers of students involved, the various learning abilities of these students, and the management skills required to co-ordinate the lessons successfully. However, the achievements of students as they engaged in with the mathematical problems were significant. From the data presented, students constructed viable solutions to the various mathematical problems that they were presented with, that were often quite different to those a teacher might have expected. Some students engaged in constructive discussion and debate about appropriate solutions that might be employed while solving problems. Students displayed enthusiasm and eagerness for mathematics not usually typical of them according to teachers. In Emily's case, she explained that her students pressured her into allocating more time for mathematics than she would have done during a normal day. Students that were most successful, and that had a clear understanding about any problem solved or a strategy used, were those in a classroom whose teacher had fully embraced the emergent perspective on constructivism.

7.3 Recommendations

This research explored the teaching of mathematical problem-solving from a constructivist perspective and now, mindful of the context in which this study was conducted and aware of its limitations, a number of recommendations will be identified arising from the conclusions of the research. These recommendations will include implications for theory formed as teachers endeavoured to move from a theory of learning to a theory of teaching, implications for policy and finally, implications for practice.

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7.3.1 Implications for theory

In the following, the most important theoretical contributions that this study has made to the teaching of mathematical problem solving from a constructivist perspective will be summarised. As teachers began to interpret a theory of learning and determine the implications for classroom teaching, significant issues arose which have theoretical implications. Polya's (1945) heuristic was utilised as an initial starting point as teachers began to re-orientate the cultures of their classrooms but there were difficulties with the engagement of students in all four stages. In particular, the broad nature of the activities that are suggested for stage four and in particular generalising from the experience is particularly difficult at primary level. During this study, teachers that engaged students in explaining their solutions to their classmates found that such experience reinforced all students' understanding of the method utilised to solve the problem. Research into the activities associated with each particular stage is necessary as Polya was a research mathematician that endeavoured to formalize the problem solving process and therefore care is needed in interpreting it for use with students learning from an emergent constructivist perspective. Furthermore, the type of mathematical problem that is selected for use with Polya's (1945) heuristic can be problematic. All four stages place significant demands on the students trying to solve the particular problem and therefore, as a prequel to Polya's (1945) four stages teachers should engage students in whole class debate and discussion to revisit key difficulties and challenges encountered when utilising Polya's (1945) heuristic previously. This is recommended due to the intrinsic value witnessed by teachers in such whole class debate and discussion at the end of the process.

Literature surrounding the emergent perspective on constructivism is vague on the implications of this learning theory for the mathematical problem classroom. Although Windschitl (1999) presents key features of classrooms that are coordinated from a constructivist perspective, it is prudent to make further recommendations about specifically what applies to the mathematical problem solving classroom. General activities associated with endeavouring to facilitate

learning from an emergent constructivist perspective are presented by Cobb and Yackel (1996) and Stephan and Cobb (2003) but further elaboration is specifically required from a mathematical problem solving perspective. In particular, the emergent perspective on constructivism does not address how teachers might ensure that both cultural and social processes and the efforts of the individual are effectively managed in the mathematical problem solving situation. This is particular to mathematics because of the strong knowledge base that has existed in the science for centuries. Teachers that endeavoured to implement principles associated with a classroom organised from a constructivist perspective found it difficult to determine how to manage and guide students in constructing mathematical knowledge consistent with culturally acceptable mathematical knowledge. This research has shown that successful efforts at facilitating learning from a constructivist perspective include the careful selection of mathematical problems. The selection of these problems is critical and will wholly depend on the knowledge of the particular students involved. Good questions are those that promote debate and discussion and allow the student an appropriate amount of choice in terms of methodology in their attempts to solve them. It is the professional judgement of the teacher that will be called upon in this regard.

7.3.2 Implications for policy

This particular study focussed on the implementation of constructivist principles espoused by the primary curriculum (Government of Ireland 1999a; 1999b) and therefore implications for national policy as a result of this study are highlighted. Although constructivism is a key feature of how the curriculum suggests teachers should approach mathematical problem solving, it is quite vague on the particular perspective of constructivism espoused. From a researchers perspective it becomes clear that the primary curriculum (Government of Ireland, 1999a; 1999b) reflects the principles of the emergent perspective, however from a readers perspective little background is offered to place it's centrality to the curriculum in context. Furthermore, as has been revealed by this study, the implications of facilitating learning from an emergent constructivist perspective are difficult for teachers. The curriculum

(Government of Ireland, 1999a; 1999b) offers little considerable practical advice to teachers about the employment of the constructivist learning theory and indeed, the presentation of the content of the curriculum in clearly defined units places significant restrictions on teachers engaging students in learning from an emergent perspective.

When the Primary Curriculum (Government of Ireland, 1999a; 1999b) was being introduced in schools, teachers were afforded in-service education by the government to facilitate this introduction. This research has established that successful in-service needs to be classroom based with particular emphasis placed on prolonged periods of classroom support which is consistent with current literature (Loucks-Horsely, Hewson, Love and Stiles, 1998). Teachers involved in this study reported that in-service in relation to the teaching of mathematical problem solving was unsatisfactory. Indeed, the NCCA's own review of the implementation of the Primary Curriculum found that teachers have difficulty with engaging students in cooperative learning which is a key feature of the emergent perspective of constructivism. Furthermore, Snyder, Lippincott and Bower (1997) suggest that the most effective method employed in the professional development of beginning teachers is a practice oriented model where participants devise plans, implement them and reflect upon what happens as a result which is not a feature of professional development in the Irish primary school situation. Therefore, when beginning teachers are inducted into schools and their profession and as constructivist theory is central to the mathematics curriculum, care must be taken to ensure appropriate support is provided as teachers attempt to engage children in learning from a constructivist perspective.

Current government policy in relation to class size has particular implications for engaging students in learning from a constructivist perspective. This study has proven that facilitating learning from a constructivist perspective is a significant challenge in classrooms with a high pupil teacher ratio. The current pupil teacher ratio of 28:1 (Government of Ireland, 2009) will ensure that pupils will be taught in large groups. Windschitl (1999) has explained that learning situations organised from an emergent perspective are highly charged and full of

energy. Therefore, current policy places significant burden on the Irish primary teacher in the implementation of the curriculum as intended and therefore, it should be examined.

7.3.3 Recommendations at pre-service and in-service level

Teaching mathematical problem solving from a constructivist perspective is fundamentally different to traditional instruction. Consequently, significant experiences in both constructivist teaching practices and mathematical problem-solving are warranted at student teacher level. Furthermore, this research has shown that the teaching of mathematical problem-solving itself is not explicit and, therefore, that an understanding of the importance of attaining autonomy in mathematics through problem solving and higher order mathematical processes is desirable. The following are recommended

- Teacher education programmes should endeavour to ensure that future teachers experience learning from a constructivist perspective.

- Student teachers should experience a constructivist approach to learning. They should test, develop, justify, argue and debate several strategies and solutions for solving mathematical problems in collaboration with their peers.

- Student teachers should study and examine a variety of constructivist teaching video episodes illustrating not just the teaching of mathematical problem-solving but all aspects of the mathematical strands, with particular reference to
    - appropriate relationships between students and teachers when engaged in constructivist teaching and learning
    - strategies for engaging mathematics students in successful collaboration
    - the development of sociomathematical norms in the classroom.

- Following the development of an understanding of constructivism and its implications for the mathematics classroom, student teachers should practice facilitating mathematical lessons from a constructivist perspective with small

groups of primary students to address classroom management difficulties and allow them to develop their own strategies for teaching.

- Student teachers should reflect on their employment of constructivist teaching practices and identify various successes and failures in their attempts to refine their repertoire of skills.
- Teacher education programmes should ensure that the teacher education students' own personal knowledge of mathematics is appropriate, so that the challenges of a constructivist classroom can be faced with confidence.

In their efforts to implement curriculum, primary teachers are afforded in-service education by the Department of Education and Science. Teachers also participate in in-service education by choice and, therefore, any in-service should focus specifically on helping teachers understand and utilise various teaching strategies including constructivist strategies. While many of the recommendations made for pre-service teacher education may be applicable to practicing teachers, the following are particularly relevant:

- Teachers should participate in further education in modularised courses leading to qualification at certificate, diploma or degree level. The course on constructivism utilised for the purposes of this research may be useful here.

- Teachers should examine teaching instances conducted from a constructivist perspective, in order to see primary mathematics students engaged in mathematical problem-solving from a constructivist perspective and subsequently debate these instances and identify their appropriateness for their own classrooms.

- Teachers should be supported over an extended period in their attempts to implement constructivist teaching methodology in their classrooms and, particularly, have mathematical lessons from a constructivist perspective modelled in their own classrooms.

- Teachers should collaborate with colleagues in an endeavour to highlight best practice in conducting mathematical lessons from a constructivist perspective and in the analysis and selection of appropriate mathematical problems.

7.3.4 Instructional Implications

Throughout the period of research the classrooms of the participating teachers reflected high levels of engagement with children purposefully engaged in solving mathematical problems. Analysis of all the teaching practices observed and the problem-solving endeavours of students suggests a number of instructional implications for effective practice.

- It is necessary to utilise a framework for mathematical problem-solving and make this framework explicit for children. Such a framework could resemble Polya's (1945) four-stage problem-solving procedure

- Teachers should model a problem solving procedure, with particular emphasis on stages one, two and four, in order to highlight the relevance of these stages to students of mathematics

- Teachers should provide appropriate support for students as they engage with mathematical problem-solving.

- The teachers should provide encouragement to students, selecting and applying a variety of different mathematical problem-solving strategies.

- The teacher should display, discuss and debate these mathematical problem-solving strategies in whole class situations following the problem-solving exercise.

- The teacher should use mixed ability grouping arrangements when structuring mathematical problem-solving lessons from a constructivist perspective to ensure that all students have experience in holding a variety of roles and responsibilities within the group.

7.4 Implications for further study in the research area

While the present study has provided invaluable insights into the teaching of mathematical problem-solving using constructivist methodology in the classroom, much research remains to be conducted concerning the employment of constructivist methodology with the entire mathematics curriculum and with other curricular areas. A review of the current research offers the following possibilities:

- The present research question could be replicated on a much larger scale and could focus on the teaching of specific mathematical concepts from across the curriculum strands, including early mathematical activities, number, measures, shape and space, data, and algebra.

- Research could provide a selection of appropriate mathematical problems that would facilitate the teacher in starting to use a constructivist approach to mathematical problem-solving.

- The implications of class size in exploring mathematical problem-solving from a constructivist perspective needs to be explored.

- All mathematical strands and strand units should be explored from a constructivist perspective to determine the optimum starting point for classroom teaching and learning.

- Appropriate structures for supporting the exploration of mathematical problem, solving from a constructivist perspective within the classroom need to be explored and identified.

- A longitudinal study of students as they progress through the primary school, with specific emphasis on their experience of constructivist learning across the mathematics curriculum, should be undertaken.

- The use of mixed ability grouping practices in the teaching of mathematical problem-solving should be investigated.

- The utilisation of general constructivist principles across the primary curriculum should be examined.

7.5 Concluding Statement

This thesis has examined constructivist approaches to teaching mathematical problem-solving in the senior primary classroom. It has followed five teachers as they attempted to implement reform pedagogy in large Irish primary classrooms. The research provided these teachers an opportunity to participate in exploring constructivism and its implications for primary mathematics instruction. By exploring constructivism from both historical and philosophical perspectives, participating teachers began to appreciate the intrinsic value of constructivist teaching strategies for primary students' mathematical development. The challenges that were faced were difficult and many, but given the timeframe of the research and the nature of classrooms and change, particular evidence emerged to support the implementation of constructivist practices in the mathematical problem-solving classroom. Successful problem solving lessons conducted from a constructivist perspective revealed the value in having primary students of mathematics debate, experiment with, and select a variety of problem-solving strategies in collaboration with one another to solve mathematical problems. The voices of both teachers and students contributed to a rich source of data that helps to illustrate the challenges associated with implementing curricular policy in the primary classroom, and the conclusions and recommendations of the research will help inform future teacher development and education programmes as we endeavour to implement new pedagogies.

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