A PICTURE IS WORTH A THOUSAND WORDS: INSIGHTS INTO GRAPHICACY SKILLS OF PRIMARY PROSPECTIVE PRESERVICE TEACHERS

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Graphs are an integral component of primary and secondary level mathematical experiences as a part of the probability and statistics strand. A review of curriculum implementation in primary mathematics revealed that, relative to other areas of mathematics, teachers found the data strand least useful in the planning for and teaching of mathematics. Furthermore, it was ascertained that data received the least classroom attention leading to the recommendation that research is needed to help 'develop guidance to support teachers in implementing this aspect of the Mathematics Curriculum' (NCCA, 2005b, p.7). This paper presents an analysis of the content knowledge skills of 456 preservice teachers in the area of graphical representations in an effort to identify their professional development needs. Participant responses on a survey of statistical knowledge are analyzed and discussed in an effort to identify strengths and weaknesses in graphicacy skills. Tasks were derived from the OECD Programme for International Student Assessment (PISA, 2003) and the National Assessment of Educational Progress (NAEP), the largest nationally representative assessment of school mathematics in the United States. Analysis of the data indicates wide-ranging difficulties with graph construction, graph selection, and understandings of data type.

INTRODUCTION

Understanding of statistics is becoming an increasingly important skill in today's society. The importance of enabling people to become statistically literate in an information-laden society is highlighted in national and international educational initiatives and curriculum documents (e.g., UNESCO, 1990; European Commission, 1996; National Council of Teachers of Mathematics, 2000). Graphical representations display quantities of measured data geometrically and are used to describe, summarize and explore data through the combined use of numbers, words, and pictures. It was nearly 200 years ago that William Playfair first employed the use of graphs in examining data. Since then, graphs have become pervasive for the processing of information and in the interpretation and analysis of data (Tukey, 1977; Pittenger, 1999).

The important function of graphs as tools in the making of meaning has been identified by a number of researchers (Cobb, 1999; Lajoie, 1993; Scaife & Rogers, 1996; Zhang, 1997). Tufte (1983) describes excellence in statistical graphics as consisting of "complex ideas communicated with clarity, precision, and efficiency". The use of graphs allows observation of trends that occur in the data, trends that may be missed with the use of descriptive statistics. As Tukey (1977) noted, "the greatest value of a picture is when it forces us to notice what we never expected to see." Tufte (1983) describes graphics as *revealing* data and states that graphics can be superlative to statistical computations in revealing information about data.

THEORETICAL PERSPECTIVE

Modes of representation Across multiple disciplines, there have been calls for greater attention to be paid to the use and combination of symbols in mathematical representational systems (Kaput, 1987). Specifically, the translation processes involved in moving between modes of representations is one fundamental use of symbolism that has been overlooked (Janvier, 1987). The importance of translation processes has been asserted by several researchers, notably by Lesh (1979) and Burton (1979) who assert their importance in problem solving, and Bell (1979) who declares them as playing a crucial role in mathematical modeling. In the case of statistics, the establishment of connections among modes of representation of data is critically important for developing understanding of data (Bright & Friel, 1998; Janvier, 1987). Similarly, the field of science depends very much on the transformation of nature into mathematical representations, within science contexts these transformations are referred to as transcriptions (Roth & McGinn, 1998). Shah & Hoeffner (2002) in their review of graph comprehension research identified the benefits of engaging learners in the activity of translating between representations. Experiences in translating between representations, they contend, may enhance the ability to see connections between quantitative information and the associated (graphical) visual features thus developing graph comprehension skills.

Figure 1, adapted from Janvier (1987) illustrates the translation processes involved in moving between three modes of representation: verbal descriptions, tables, and graphs; and identifies six different processes utilized when translating between representations. On examination of any pair of representational variables, there are two translations that facilitate movement between the representations, the appropriate translation to use being determined by the "target point of view" (Janvier, 1987, p. 29). For example, when considering the representational variables: table and graph, if we wish to translate from a table to a graph (table → graph) then the translation process of interest is *plotting*. If however, we wish to translate in the other direction from graph to table (graph → table) then the translation process of interest is *reading off*.

To / **From**	Situations, Verbal Descriptions	Tables	Graphs
Situations, Verbal Descriptions		Measuring	Sketching
Tables	Reading		Plotting
Graphs	Interpretation	Reading off	

Figure 1: Translation processes for moving between modes of representation

Within the context of primary statistics education, the focus on engaging students in the process of statistical investigations necessitates a complete cycle of translation processes from situation descriptions that identify variables of interest and the measurement of those variables (situation descriptions → measuring), to the construction of representations (table → graph), followed by analysis and verbal descriptions of underlying patterns (graph → verbal descriptions), which can in turn be related back to the situation descriptions.

Research examining the translation processes *table → graph* and *graph → table*

Most cognitive research on translations and graphical representations has focused on graph comprehension - where comprehension refers to reading and interpreting graphs. Four critical factors that affect graph comprehension have been identified by Friel, Curcio & Bright (2001) as purposes for using graphs, task characteristics, discipline characteristics, and characteristics of the reader. Despite what studies have contributed in terms of identifying factors which should be considered when making graphs, graph comprehension has been shown to be a complex and demanding activity fraught with systematic errors (Carpenter & Shah, 1998; Leinhardt et al., 1990; Shah & Carpenter, 1995). Work pertaining to graph comprehension at the school level has made valuable contributions in terms of identifying aspects of statistical literacy in the interpretation of graphs. Research carried out on school students' interpretations of graphs (Curcio, 1987) culminated in a commonly used typology of types of graph interpretation: *reading the data, reading between the data, and reading beyond the data.* Recommendations for the sequence of instruction for the introduction of graphs at schools have been made by Friel et al (2001). For a detailed and comprehensive review of the research on graph comprehension refer to the review of graph comprehension research carried out by Shah & Hoeffner (2002) and Friel et al (2001).

In contrast, much less work has been carried out on graph construction. Friel et al. (2001) use the term *graph sense* to refer to the skills associated with reading, interpreting, and constructing graphs in addition to choosing the graph best suited to particular situations. This construct of graph sense provides a framework within which certain behaviors associated with graph sense are identified. Within the construct of graph sense, they identify three abilities which directly relate to graph construction: (1) To recognize the components of graphs, the interrelationships among these components, and the effect of these components on the presentation of information in graphs; (2) To understand the relationships between a table, a graph, and the data being analyzed; and (3) To recognize when one graph is more useful than another on the basis of the judgment tasks involved and the kind(s) of data being represented (p. 146).

TEACHER KNOWLEDGE AND STATISTICS EDUCATION

Over the past several decades, educational research has focused on identifying the elements that contribute to good teaching, in particular the relationship between subject matter knowledge and pedagogy. Ball & McDiarmid (1990) suggest that three outcomes derive from subject matter learning: *substantive knowledge of the subject* (understanding the information, ideas and concepts associated with a specific field), *knowledge about the subject* (being knowledgeable about the fundamental activities of the field) and, d*ispositions toward the subject* (developing preferences for particular areas of the field). Although a necessary prerequisite for teaching, subject matter knowledge is not sufficient. Pedagogic knowledge, defined as 'the most useful from of analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject in order to make it comprehensible to others' (Shulman, 1986, p.9), is also necessary. Good teaching, it is generally agreed, requires both subject matter knowledge and pedagogic knowledge, and depends on 'the capacity of *a teacher to transform the content knowledge he/she possesses* into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students' (Shulman 1986, p. 9). While this study acknowledges the role that both subject matter knowledge and pedagogical content knowledge play in teaching, the focus of the study is on the subject matter content knowledge (Shulman, 1986) of preservice teachers. Referring back to the framework posited by Janvier (1987) this study will focus exclusively on the translations: *table* $\rightarrow$ *graph* and *graph* $\rightarrow$ *table.*

The influence of subject matter knowledge on everyday pedagogical decisions, such as posing questions and setting tasks, has been established (Grossman, Wilson & Schulman, 1989; Hashweh, 1987; Thompson, 1984). The importance of teachers possessing a deep understanding of specific mathematical concepts and ideas (Ma, 1999) cannot be overemphasized. Poor teacher mathematical content knowledge often results in an emphasis on the richness and meaningfulness of mathematical concepts being replaced by a focus on engagement in mathematical activities without the corresponding focus on meaning (Heaton, 1992; Putnam, 1992). The importance of attending to preservice teachers' statistical knowledge, in particular, is becoming increasingly evident. As Heaton and Mickelson (2002) note:

> If statistical education is to be addressed seriously in elementary education … specific focus needs to be placed on the learning of teachers … We cannot seriously attend to children's understanding of statistics without simultaneously attending to teachers' understandings.

A number of studies have examined how preservice and practicing teachers read and interpret graphical representations. Teacher attitudes about graphs have been found to be positive and teachers are confident when it comes to teaching graphs. This confidence was evident in a study of preservice teachers carried out by Watson (2001). Administration of an instrument designed to profile teachers' competence and confidence to teach probability and statistics found that pre–service teachers reported greater confidence in teaching graphical representations than other statistical concepts (median, average, data collection). This confidence relating to graphical representations was also a finding of a study of secondary preservice teachers (Gonzalez & Pinto, 2008). The authors found that pre-service teachers underestimated the complexities of the mathematical ideas fundamental to statistics education, and these same teachers who appeared confident in their understandings of graphical representations were identified as having limited knowledge and treatment of graphical representations (Gonzalez & Pinto, 2008). In contrast, the presentation of media graphs to preservice teachers demonstrated critical thinking skills in the interpretation of data presented on the graphs (Monteiro & Ainley, 2006). It should be noted that the context of reading *media* graphs, the authors contend, may be different to school graphs and both contexts may draw on different kinds of knowledge when interpreting the data. Taking this into consideration, it is difficult to ascertain to what extent the critical thinking skills relating to media graphs as demonstrated by pre-service teachers in this study extend to school-graphing contexts.

Other studies have examined how pre-service teachers work with graphical representations within pedagogical contexts. A study of preservice teachers carried out by Heaton and Mickelson (2002) provides some insight into the processes that occur between data collection and data construction. The authors observed that graph construction often became the endpoint of statistical investigations, with the predominant focus being on technical aspects of graph construction. The preservice teachers tended not to engage children in reasoning about the data through examination of graphical constructions, nor did they use the constructed graphs to coordinate a refocus on the initial purpose of the statistical investigation i.e. the research question. A similar study of elementary preservice teachers carried out by Leavy (2006) engaged participants in two semester long statistical investigations. The study revealed the tendency to focus on descriptive statistics when examining data. Participants did not recognize graphical representations as important tools in supporting the description and comparison of distributions of data. Furthermore, those participants who created graphs, at the outset of the study, did not depict the individual data values. Rather they used the graphs to report descriptive statistics. Reliance on descriptive statistics as opposed to graphs was persistent throughout the study. Leavy argues that descriptive statistics are appropriate as comparative measures, absolute reliance on descriptive statistics is limiting as they provide merely one perspective on the data, that of centers, and do not take into account other features of the data e.g. shape, variability.

Fewer studies have examined the skills of pre-service teachers when *constructing* graphs i.e. the *table → graph* translation, which is the focus of this study. Little is know about pre-service teachers' own practices in constructing representations from data. Examination of the translation processes of pre-service science teachers during science investigations sheds particular light on the table → graph translation (Bowen & Roth, 2005). Two different levels of difficulty were identified. The first difficulty related to pre-service teachers not knowing if a graph should be used. The second difficulty was fundamental to the construction of representations and related to not knowing how to structure the data and choose inscriptions that were appropriate for the context chose. All these studies highlight difficulties understanding the role of graphical representations within the process of statistical investigation.

METHOD

Participants

Participants were entry-level primary education students and tested in their first week of Year 1. They were given 50 minutes to complete the survey. Participation was voluntary and anonymous. Tests were administered within groups of 50. In all, 456 of 480 participants completed the survey.

Task design and administration

This was a large-scale survey administration of 12 statistical items. Five tasks relate to graphical literacy, this study reports on three of those tasks. Table 1 outlines the categorization of all five tasks, survey placement, and the number of participants administered the task. The two PISA tasks will not be discussed in this paper.

Identifier	GAISE Code	Source	Survey version	Sample size
2a	AD1	Designed for study	A	118
2b	AD1	Designed for study	B	110
2c	AD1	Designed for study	C	119
2d	AD1	Designed for study	D	113
3	AD3	Designed for study	C, D	231
4	AD4	PISA	A, B, C, D	456
5	AD4	PISA	A, B, C, D	456
6	AD4	NAEP	A, B	225

Table 1: Categorization of survey tasks

The framework used to categorize tasks is the GAISE (2005) framework outlining the *processes of statistical investigation*: formulating a research question (FQ), collecting data to answer the question (CD), analysing the data (AD). Tasks were then located within the Irish primary curriculum (DES, 1999) and the National Council of teachers of Mathematics (NCTM) *principles and standards for school mathematics*. Given the number of tasks, four versions of the survey were administered (A, B, C, D).

RESULTS

Task 1: Constructing graphs [The paperclip task]

The purpose of this task was to identify graph construction skills. Participants were presented with raw data and asked to translate the data to a graph (table → graph) using the process referred to as *plotting* (Janvier, 1987); different test versions requested the construction of different graphs (bar charts, pie charts, histograms, and most appropriate graph). Pie and bar charts were included due to their positioning within the Irish Primary School Curriculum. The histogram was chosen due to its inclusion in the secondary school curriculum and participants' familiarity with the representation. The fourth variant allowed the participant the choice of any graph to construct based on their determination of the way to best represent the data. This latter open-ended question involving an element of choice is not a common practice in Irish curricula but is a skill identified by Friel et al. in their definition of *graph sense*.

Task 2The paperclip task

The following values represent the distances (in cm) that a group of 30 students blew a paperclip. Each value represents each participants' best attempt. Graph the data using the following graph: pie chart/histogram/bar chart/most appropriate graph.

2	10	17	22	22	22	23	25	27	27
28	34	40	50	50	50	52	54	55	60
60	64	66	68	73	76	84	85	150	180

As can be seen from table 2, 34% of participants correctly constructed a *pie chart.* The traditional taught strategy was the least prevalent – only 3 of 118 participants used this strategy. The most prevalent approach was division of the circle into the total number of data values and shading the sub parts in accordance with the frequencies displayed in the data. This is not a traditionally taught strategy and has been found with children who have not been taught traditional methods of constructing pie charts (Leavy, 2007). 11% of participants accurately constructed a *histogram* (version b), a representation prevalent in secondary school. Interestingly, 26% erroneously completed a bar chart rather than a histogram. 52% accurately constructed a *bar chart* (version c). The majority of bar charts were case value charts (47%) and the remaining were frequency bar charts. 12% of (incorrect) responses involved the construction of a histogram rather than a bar chart. When permitted to construct *any graph of their choice* (version d) 40% of participants constructed a graph that accurately presented the data. Most popular were frequency line plots (12%), bar charts (12%), and histograms (12%). Pie charts (1%) and line plots (1%) were least popular. The most prevalent response in this category, 36%, was the construction of a trend graph. However, a trend graph indicates that the data values are linked by time, in other words that the values have a chronological order. The presented data set has no relationship or connection with time; hence the construction of this representation is inappropriate for this data set.

	Correct	Incorrect
Version a: Pie chart	34%	66%
Version b: Histogram	11%	89%
Version c: Bar chart	52%	48%
Version d: Most appropriate	40%	60%

Table 2: Performance on the graph construction task

Taken in its entirety, less than one third (30%) of participants provided a complete and accurate response to the task. Furthermore, the number of incomplete or non-responses accounted for 36% of all responses. The bar chart was the representation constructed with most success. However, confusion existed relating to bar charts and histograms with almost 10% of all participants confusing the representations. The low popularity of pie charts on versions d, when taken in conjunction with the low success rates with pie charts on version a, indicates that pie charts pose considerable challenges for preservice teachers – which in turn raises questions about their suitability for inclusion on the primary curriculum.

Task 2: The relationship between data type and graphical representations [The ice cream task]

The purpose of this task was to ascertain if an understanding exists of which graphs are appropriate for display of categorical data. Relating data type and associated graphical representation is important in that certain data types can be represented only on particular graphs. Otherwise, what emerges is a skewed, and at times, inaccurate representation of the data. The options presented were derived from commonly used graphs, in NCTM and GAISE documents, for use with preservice teachers. The GAISE document defines teacher knowledge necessary to complete the task as 'Teacher possesses conceptual/relational understanding of how certain data types can be represented only on specific graphs'. The task addresses the NCTM objective 'In grades 3–5 all students should recognize the differences in representing categorical and numerical data.' There is not an equivalent objective in the Irish Primary Curriculum.

Task 2: The ice cream task

The following are responses of 12 children when asked to identify their favourite ice cream flavour:

Mint	Strawberry	Banana	Mint	chocolate	chocolate
chocolate	Banana	Mint	Vanilla	Banana	Banana

Please circle each of the following graphs that may be appropriate to use to graph the data:

pie chart	histogram	bar chart	bar-line	box and whisker plot
pictogram	line plot	dot plot	ogive	stem and leaf plot

Given the lack of attention to data types and their associated graphs in the Irish Primary curriculum, it is not surprising that 9% of participants correctly identified the graphs suitable for display of categorical data. 65% over estimated the number of graphs suitable for displaying categorical data. It should be noted that given the limited menu of graph options presented in Irish secondary school curricula, there is the possibility that participants were not familiar with all options presented in the task.

Task 3: NAEP election data

An election involving four candidates for mayor has been held. Of the following, which is the best way to present the percentage of votes each candidate received?

A) Circle graph

B) Line graph

C) Box plot

D) Scatterplot

E) Histogram

Please justify your answer:

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The purpose of this task was to ascertain whether participants could choose a suitable graph to illustrate patterns in data. The task is a released NAEP task from the 2005 administration to 12th grade students. Within the NAEP classification, this item is classified as *low complexity* as it 'relies heavily on the recall and recognition of previously learned concepts and principles. Items typically specify what the student is to do, which is often to carry out some procedure that can be performed mechanically. It is not left to the student to come up with an original method or solution'.

Although this was one of the better-answered questions, with 48% of participants getting the correct solution, more than half answered the item incorrectly. The results are particularly worrying when the results are compared with those of American 12th graders who answered the same task. 60% of US 12th grade students chose the correct graph to represent the data – a significantly higher number than pre-service teachers in this sample. One reason which may account for low performance on this task is that the Irish statistics curricula generally prescribe the graph to be constructed – classroom instruction then focuses on the construction and interpretation of the graph. Students are rarely, if ever, involved in discussion regarding the selection of an appropriate graph to display data. This task focused specifically on determining if the respondent could select the graph most suitable to illustrate aspects of a situation. The reasoning skills involved in making these decisions are poorly developed in Irish curricula.

DISCUSSION

The purpose of this study was to examine the graphicacy skills of primary pre-service teachers. The study was carried out on entry to the College of Education and prior to any instruction in mathematics education pedagogy. In this sense, the content knowledge understandings on exit from secondary school were the focus of study. The five tasks administered to participants can be categorized in terms of difficulty level by reference to the graph complexity organizer posited by Friel et al (2001) in addition to the appearance of the statistical concepts in school curricula (NCTM, 1999; DES, 1999).

The first two tasks, the paperclip task and the ice cream task, are of comparable difficulty as both draw on content knowledge understandings expected from primary level children. Performance on graph construction in task 1 is extremely concerning, with only 30% correct responses across all versions of the task, considering the extent of participants exposure to the representations over their school lives. Task 2 showed the lowest success rate with only 9% correct responses in determining which representations best suit categorical data. These results highlight the need for Irish curricula in the area of data handling and statistics to better reflect best practices evident in international curricula. The results highlight the need for greater emphasis to be placed on the nature of data, in particular numerical and categorical distinctions. Furthermore we need to question the suitability of pie charts in primary level curricula in light of the fact that (a) two-thirds of pre-service primary teachers themselves demonstrate profound difficulties with these representations, and (b) less than 1% used this representation when provided with a menu of graph options. The critical need to highlight the distinctions between graphs and their uses, particularly the bar chart versus histogram distinction, is also clearly evidenced in this study.

The NAEP Election Data task, task 3, was defined as a 'low complexity' task as it relies on recall of previously known facts. Almost half of the responses correctly identified a circle (pie) chart as the representation most appropriate for the presentation of percentage of electoral votes. It is very disconcerting that pre-service primary teachers fared considerably less well than 12th grade American students. Moreover, the sample of Irish pre-service teachers represents a relatively high achieving academic group as compared to a random sample of US students. The fact that 52% incorrectly responded to this task highlights poor ability to discern the functionality and use of particular graphs, calling to question the statistical literacy of the pre-service teachers in this study.

In conclusion, the poor performance on graphicacy tasks calls into question the statistical literacy of pre-service primary teachers. Participants in this study performed poorly on basic and important skills represented in school mathematics. The results of this study have implications for the design of instructional activities and curricula at the primary and secondary level. The results presented here also highlight the need for focused research involving integrated cycles of instructional design and analysis of student understanding in an effort to better support the development of students' statistical literacy, reasoning and thinking at all levels.

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