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PAPER DRAFT

PROSPECTIVE PRIMARY TEACHERS UNDERSTANDINGS OF GRAPHICACY

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This paper reports on the statistical knowledge of 456 entry-level, prospective teachers relating to data representation. We report on the responses of participants to three items sourced from the OECD Programme for International Student Assessment (PISA) and the National Assessment of Educational Progress (NAEP). We discuss the implications for the teaching of statistics in classrooms and provide recommendations to improve the statistical experiences of future teachers, and in turn the children they will teach.

INTRODUCTION

The incorporation of data handling into elementary mathematics curricula has sharpened the focus on developing statistical literacy, thinking and reasoning. We want students to construct rich conceptual understandings of statistics, develop connections between procedures, concepts and representations, and engage in dialogue and discourse around the process of statistical investigation. Supporting the construction of these types of competencies requires that teachers themselves have rich connected understandings of statistics and also requires that many teachers teach statistics in ways counter to how they themselves were taught. Arising from research on teacher's mathematical knowledge and its role in teaching and the interplay between mathematics and pedagogy, Ball has developed a theory of mathematical knowledge for teaching (MKT) (Ball, 1999; Hill, Rowan & Ball, 2005). This type of knowledge includes both mathematical knowledge common to those working in diverse professions (Subject Matter Knowledge, SMK) and the mathematical knowledge that is specialized to teaching (Pedagogical Content Knowledge, PCK). While both components have been demonstrated to contribute to good teaching practices, this paper focuses on one of the dimensions: Subject Matter Knowledge. This study explores prospective elementary teachers' skills of representing and interpreting data, skills that may in turn support their teaching of data in primary level classrooms.

Graphical representations as tools in the making of meaning has been identified by a number of researchers (Lajoie, 1993; Scaife & Rogers, 1996). Tufte (1983) describes excellence in statistical graphics as consisting of "complex ideas communicated with clarity, precision, and efficiency". The use of graphs allows observation of trends that occur in the data, trends that may be missed with the use of descriptive statistics. As Tukey (1977) noted, "the greatest value of a picture is when it forces us to notice what we never expected to see." Tufte (1983) describes graphics as *revealing* data and states that graphics can be superlative to statistical computations in revealing information about the data. Few studies examined how teachers conceptualize graphical representations. We do know, however, that graphical construction often became the endpoint of statistical investigations with the predominant focus on technical aspects of graph construction rather than using graphs as a tool to illustrate patterns and trends in data (Heaton and Mickelson, 2002). Furthermore, studies indicate that teachers may not integrate graphical analysis and interpretation components into statistical units in a pedagogically sound and conceptually appropriate manner. These findings lead to the primary focus of this study: what understandings and skills do prospective teachers possess in terms of selecting and using graphs to communicate aspects of data?

METHODS

The study involved the administration of a statistical tasks to 456 entry level pre-service primary teachers. The tasks used were designed to elicit models of understanding of certain statistical phenomena. The overlying framework used to categorize tasks is the *Guidelines for Assessment and Instruction in Statistics Education (GAISE) framework* (Franklin et al., 2007). Each task was located within the GAISE framework, situated within primary curriculum frameworks, and the type of teacher understanding necessary to complete the task was described. We report on the outcomes arising from three tasks relating to graphical literacy.

The Indonesia Task (PISA): The purpose of this open constructed-response item task, was to ascertain whether participants could choose a suitable graph to illustrate patterns between variables in data. 20% of prospective teachers constructed a graph that represented the uneven population distribution: Scatterplots (8%), double bar charts (4%) and a variety of alternative representations (8%) accounted for the correct responses (see image 1). 35% did not attempt the task, with many indicating that they did not know how to represent the variables. Over a quarter managed to represent each variable separately on either a bar chart (15%) or a pie chart (12%) but failed to show the relationship between variables. 8% constructed trend graphs incorrectly implying some relationship between the data and time. 8% of responses were incomplete – analysis of these responses indicated multiple failed efforts to adequately display the relationships in the data.

Indonesia lies between Malaysia and Australia. Some data of the population of Indonesia and its distribution over the islands is shown in the following table:

Region	Surface area (Km2)	Percentage of total area	Population in 1980 (millions)	Percentage of total population
Java/Madura	132 187	6.95	91 281	61.87
Sumatra	473 606	24.86	27 981	18.99
Kalimantan (Borneo)	539 460	28.32	6 721	4.56
Sulawesi (Celebes)	189 216	9.93	10 377	7.04
Bali	5 561	0.30	2 470	1.68
Irian Jaya	421 981	22.16	1 145	5.02
TOTAL	**1 905 569**	**100.00**	**147 384**	**100.00**

One of the main challenges for Indonesia is the uneven distribution of the population over the islands. From the table we can see that Java, which has less than 7% of the total area, has almost 62% of the population.

Source: de Lange and Verhage (1992). Used with permission

Question 1: INDONESIA

Design a graph (or graphs) that shows the uneven distribution of the Indonesian population.

Figure 1: Indonesia Task (PISA)

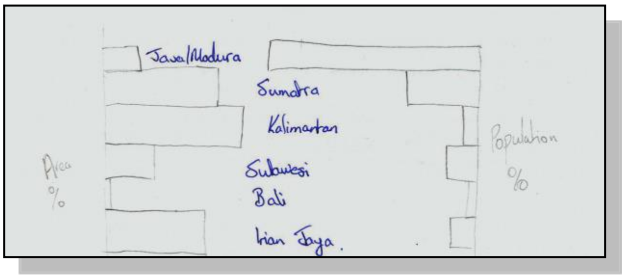

Image 1

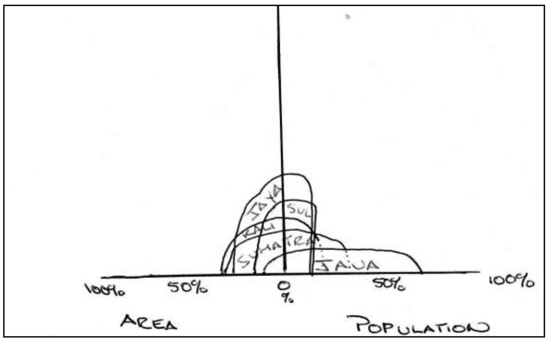

Image 2

The Zedland Postal Task (PISA): The purpose of this multiple-choice task was to ascertain whether participants could choose a suitable graph to illustrate patterns in data. 18% selected the correct option of graph c, a 'step graph'. Graph D accounted for the majority of incorrect responses (52%). One reason that may account for the popularity of this response has to do with scaling issues. This graph, as compared to graphs a-c, facilitates observation of the trend for the 0-500 grams. In addition the scale on the x-axis is similar to the categories presented on the table of data values, a situation that may have persuaded participants if examining surface features of the problem. Graph B was a more popular choice than the correct option (21%). Examination of this graph reveals that it has many of the features that participants might expect to find in a graph – a series of points joined by a curved line. However, selecting points along the line results in prices that are not 'available' at the post office – thus the graph does not accurately reflect the pattern presented on the table of data.

Weight (to nearest gram)	Charge
Up to 20 g	0.46 zeds
21 g – 50 g	0.69 zeds
51 g – 100 g	1.02 zeds
101 g – 200 g	1.75 zeds
201 g – 350 g	2.13 zeds
351 g – 500 g	2.44 zeds
501 g – 1000 g	3.20 zeds
1001 g – 2000 g	4.27 zeds
2001 g – 3000 g	5.03 zeds

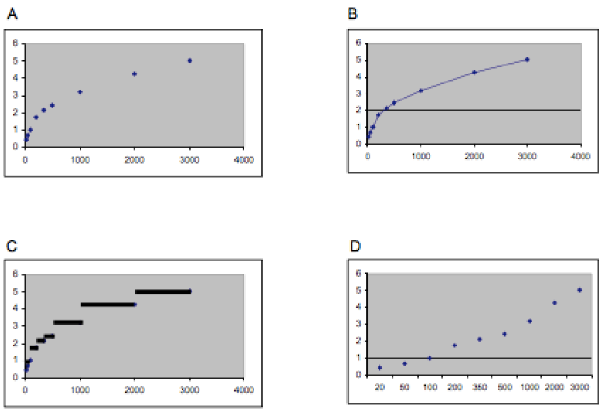

Figure 2: Zedland Postal Task (PISA)

The Election Task (NAEP): The purpose of this task was to ascertain whether participants could choose a suitable graph to illustrate patterns in data. The item is classified as *low complexity* (NAEP). Although this was one of the better-answered task, with 48% of participants providing the correct solution, more than half answered the item incorrectly. The results are particularly concerning when compared with those of American 12th graders who answered the same task. 60% of US 12th grade students chose the correct graph to represent the data – a significantly higher number than prospective teachers in this sample.

> **An election involving four candidates for mayor has been held. Of the following, which is the best way to present the percentage of votes each candidate received?**
>
> **A) Circle graph**
> **B) Line graph**
> **C) Box plot**
> **D) Scatterplot**
> **E) Histogram**
>
> **Please justify your answer:**
> ____________________________
> ____________________________
> ____________________________
> ____________________________

Figure 3: Election Task (NAEP)

FINDINGS AND DISCUSSION

Poor performance on the PISA tasks indicates lack of experience reasoning about graphs in unfamiliar and novel contexts. Prospective teachers did not appear to have adequate exposure to a range of graphical representations to facilitate exploration and comparison of distributions of data. This highlights the critical need to expand the scope of data visualization approaches by expanding the repertoire of graphs we teach at primary and post-primary level. Another plausible explanation for the low proportion of correct responses may lie in preservice teachers' belief structures and experiences relating to graphs. One suggestion is that they did not comprehend the functionality of graphical representations; in other words they did not understand the advantages afforded by constructing a graph. Participants' secondary level experiences with graphs were procedural-based with the emphasis placed on 'how to' with little attention placed on 'when to' or 'why.' Hence, the current focus on the production of graphs needs to broaden to include a focus on the functionality of graphs. Attention needs to be focused on developing understanding of the function of graphs as communicating data. Experiences need to be provided wherein the use of graphs within investigative contexts arises out of the need to display the data collected and these graphs serve as tools to explore patterns and trends in data. These experiences choosing graphs to display data will reveal the relative affordances of different graphs in terms of the types of data they display, the tendency to reveal/hide individual data values, and the emphasis placed on variability.

References

Ball, D. L. (1999). Crossing boundaries to examine the mathematics entailed in elementary teaching. *Contemporary mathematics*, *243*, 15-36.

Heaton, R.M. & Mickelson, W.T. (2002). The learning and teaching of statistical investigation in teaching and teacher education. *Journal of Mathematics Teacher Education*, 5, 35-59.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*, 371–406.

Lajoie, S.P. (1993). Computer Environments as Cognitive Tools for Enhancing Learning. In: Lajoie & Derry (Eds.) *Computers as Cognitive Tools* (Lawrence Erlbaum Associates, London).

Scaife, M., & Rogers, Y. (1996). External Cognition: how do graphical representations work? *International Journal of Human-Computer Studies*, *45*, 185-213.

Tukey, J.W. (1977). *Exploratory Data Analysis* (Addison-Wesley Publishing Company).

Tufte, E.R. (1983). *The visual display of quantitative information* (Cheshire: Graphics press).